UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

THESIS FOR M.SC. ENGINEERING

"THE OPTIMUM DESIGN OF FLAT RECTANGULAR CONCRETE PLATES SUPPORTING LOADS IN BUILDING STRUCTURES AND TESTS ON A ONE-THIRD FULL SIZE EXPERIMENTAL MODEL"

BY

EDWARD GROYER, Pr.Eng., B.Sc. (Eng) M.I.Struct.E

A.M.(S.A.)I.C.E.(S.A.)Cons.E.

INDEX

P.T.O.

SYNOPSIS

The popularity of concrete flat plate construction in Building Structures has soared in the last 20 years but their design has been based mainly on their ultimate strength rather than their behaviour in terms of deflection and cracking at working loads. As a result their deflections have been larger than anticipated and have caused displacements and severe cracks in masonry partitions which are being supported.

In order to design any rectangular flat plate for optimum performance and economy most of the present design procedures such as the Elastic, Empirical, Yield-Line, Elastic-Plastic and Finite Difference, are reviewed, critically examined and illustrated by designing the same typical flat plate consisting of three bays in each direction for a uniformly distributed transverse load.

Comparisons are made of the cost, strength and behaviour of each resulting plate.

A model plate measuring 3000 x 4132 x 63mm thick consisting of four rectangular panels supported on 9 columns was designed by the Elastic-Plastic method to support 110mm thick walls and a live load of 2,5 KN/m^2. The model was then constructed in the Laboratory and loaded to destruction by applying four concentrated loads to the quarter-points of each panel.

The results of the test showed that the slab deflected elastically at the critical sections for superimposed loads, excluding the walls, less than 80% of the theoretical Ultimate Load. The deflection could be closely calculated by Elastic Analysis, using the full uncracked section stiffness for loads less than that which causes cracking at the section for maximum positive moment, and using an effective stiffness which is intermediate between the cracked and uncracked values for higher loads but within the range of elastic behaviour of the slab.

By selecting the slab depth principally to satisfy the two criteria (a) $\frac{L}{d} \leqq 32$ (b) no mid-span cracking due to the sustained load and by distributing the reinforcement in accordance with a step-function approximation which is derived from the theoretical moments along the critical sections at the onset of yield in the reinforcement at the column edge, flat plate behaviour including its effect on masonry partitions will be optimized throughout the full loading range with a relatively small increase in construction costs.

PART 1. INTRODUCTION

"Flat Slab" structures can be described as reinforced concrete slabs which distribute transverse loads directly to a rectangular grid of reinforced concrete columns without supporting beams. Either or both column capitals and drop panels are provided.

"Flat Plate" structures are a derivative and special case of Flat Slab structures where the slab is of uniform thickness throughout and the supporting columns are either structural steel columns with bearing plates or reinforced concrete columns without capitals. It is not essential that the columns form a rectangular array but this special case will not be considered here.

In many parts of the world since the War, Flat Plate structures have become exceedingly popular, for multi-storey flats and commercial buildings, with both the Builder and the associated Professions. The reasons for this popularity can be briefly ascribed to the following:-

1. The uniform under-surface is architecturally attractive in that it allows maximum flexibility in the sub-division of internal space and the distribution of services.

2. Erection of formwork by the builder is simpler, quicker and therefore cheaper than conventional beam and slab or even "Flat Slab" construction.

3. Placing of steel reinforcement is faster, in that beams with small diameter stirrups are not required to be fixed before the slab is commenced.

4. Engineers, once they have accepted the hair-raising simplification in the design permitted by most Codes of Practice, learn to love them. This simplification extends to the setting out Drawings of Structure and Reinforcement as well as checking of steel placing.

5. The Client reaps the benefits of a more functional, flexible and possible cheaper cost of construction allied to an earlier completion date.

Inevitably /

Inevitably he will be sold on "Flat Plate" construction for all his other similar projects.

Rarely in the History of the Construction Industry has a form of Concrete Construction captured the approval and enthusiasm of all its participants as has the Concrete Flat Plate. It is however surprising that in the design, attention has been largely directed to their capacity to carry loads and that their behaviour in terms of cracking and deflection has been largely neglected. Arbitrary limitations to the ratio of $\frac{\text{span}}{\text{depth}}$ are set by most Codes in a similar manner to structural steelwork, with the intention of providing sufficient rigidity to prevent excessive deflections. The Australian Code for Concrete CA-2 1962, the ACI Building Code 518-63 and the British Code CP 114, limit the ratio to 36 irrespective of span ratio, continuity, superimposed loading, sustained loading or type of loading. Whereas plates designed to this limit were shown by research and experience to deflect initially less than the arbitrary $\frac{\text{span}}{360}$ set for all structures whether constructed in timber, steel or concrete (probably for aesthetic reasons), secondary effects increased the deflection many-fold during the course of time.

In the early 1960's flat plates which had been constructed a decade earlier began to show signs of distress in the shape of visible cracks, abnormal deflection and particularly in severe cracking of brick partitions standing on the floor. Figure A1 in the Appendix reproduced from Reference 15 shows one common mechanism of crack formation. This brought growing realisation that the deformation at working loads was as important as the ultimate strength and that even moderate deformation could cause damage to other parts of the building.

As a result of this, a comprehensive study of Flat Plate structures was begun in 1959 by the Division of
(R17) Building Research in Australia with the erection and testing of four experimental structures as well as analytical studies. The conclusions reached will be referred to at the end of the Thesis, but the following observations will explain the "excessive" deformation of Flat Plates compared to older structures :-

1. In older forms of structure such as slabs supported by deep beams on all sides which were designed to carry the full weight of masonry walls as well as the slabs, many redundancies existed in the elastic interaction of
(R37) slabs, beams and walls which increased both beam and slab stiffness. As a result computed stresses and deflections were never achieved.

2. In slender flat plates the behaviour is close to the design assumptions and all the old hidden factors of safety are gone.

Westergaard reports one flat slab structure built in World War I where the columns were 5 ft. square at 20 ft. centres resembling a Gothic Cathedral. A great deal of the load was carried by Arch action and not by bending. The modern flat plate has negligible arch action but is designed by the same method.

3. Working stresses are now twice what they were 50 years ago and with no significant change in the Elastic Modulus of Steel, the corresponding strains and deformations are twice those realised in the past.

Against the aforementioned background of practical experience and research still taking place in Australia and also other parts of the world including South Africa, the writer proposes to discuss and predict the strength and behaviour of flat plates at important stages of its life history. The principal methods of analysis or design presently used will then be critically examined in the light of these predictions. A typical example will be fully designed by each method and the relative cost of the resulting slab will be estimated.

Based on these results, a procedure for direct design will be selected which optimizes the behaviour of the plate throughout its loading history for any given cost of concrete and reinforcement. With a few small modifications this procedure can be used for optimum design of plates carrying masonry partitions.

As an illustration, a model plate will be directly designed to carry average loading including brick partitions, and then erected and tested to destruction to confirm or modify the design.

PART 2. STRENGTH AND BEHAVIOUR OF FLAT PLATES

A. Elastic Theory of Thin Concrete Plates subject to Transverse Loads.

(R34) The following assumptions are generally made provided any cracking is confined to small areas around the columns:-

1. The thickness of the plate is small compared to the dimensions between the supports and the deflections are small compared to its thickness.
2. The middle plane of the plate undergoes no lateral deformation.
3. Plane surfaces normal to the middle plane before bending remain normal after bending.
4. Normal stresses transversely to the plate can be disregarded.
5. The plate is homogeneous and isotropic elastic.

It is convenient to set up three global axes at right angles, OX, OY and OZ, so that plane XOY is horizontal and coincides with the middle plane of the plate before bending. During bending points in the XOY plane undergo small vertical displacements "w" to form the middle surface of the plate. Any point in the middle plane of the plate is completely defined by its X and Y co-ordinates which are independent. The deflection under load "w" will then be a function of x,y and the loading q, i.e. $w = f(x,y,q)$. All other surfaces in the slab parallel to the middle surface are defined by their distance z above are below it.
At any point x, y in the middle surface the following relationships exist:-

Slope of surface in X direction $= i_x = \dfrac{\partial w}{\partial x}$ ....(1)

Slope of surface in Y direction $= i_y = \dfrac{\partial w}{\partial y}$ ....(2)

Curvature of surface in the x direction =

= rate of change of slope $= \dfrac{1}{r_x} = -\dfrac{\partial^2 w}{\partial x^2}$ ....(3)

Curvature in the y direction $= \dfrac{1}{r_y} = -\dfrac{\partial^2 w}{\partial y^2}$ ....(4)

Twist of surface with respect to the X and Y axes

$$= \frac{1}{r_{xy}} = \frac{\partial^2 w}{\partial x . \partial y} \quad ....(5)$$

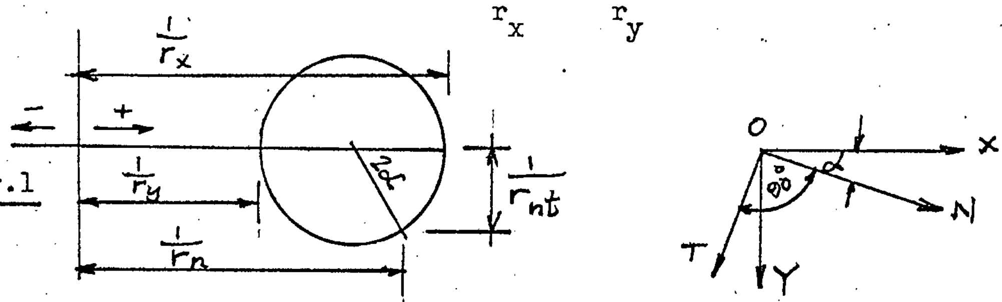
The following relationships can then be proved :-

1. At any point, the sum of the curvatures in any two directions at right angles is constant.

2. At any point the maximum and minimum curvatures, called the "principal curvatures" occur in two directions at right angles.

3. If $\frac{1}{r_x}$ and $\frac{1}{r_y}$ are the principal curvatures at any point, then $\frac{1}{r_{xy}} = 0$ and the curvature and twist in any direction ON can be obtained by a Mohr circle as shown hereunder. $\frac{1}{r_x}$, $\frac{1}{r_y}$ may be positive or negative.

Fig.1



A positive curvature denotes a surface bent convex downward thus

If both r_x and r_y are positive the surface is "synclastic"
If r_x and r_y are of opposite sign, the surface is "anticlastic" or saddle shaped. Further examples are points midway between the columns in a flat plate.

By considering an element of the slab dx x dy x h where h = slab thickness at the point x, y, the following relationships between internal moments and curvatures can be established :-

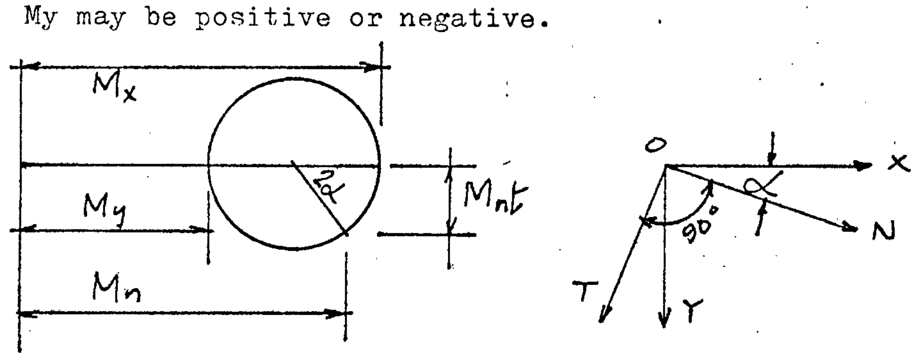
$$M_x = D\left(\frac{1}{r_x} + \nu\frac{1}{r_y}\right) = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \quad \ldots\ldots.(6)$$

$$M_y = D\left(\frac{1}{r_y} + \nu\frac{1}{r_x}\right. = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \quad \ldots\ldots(7)$$

$$M_{xy} = M_{yx} = D(1-\nu)\frac{\partial^2 w}{\partial x . \partial y}) \quad \ldots\ldots(8)$$

Again if $\frac{1}{r_x}$ and $\frac{1}{r_y}$ are the principal curvatures then Mx and My are the principal moments and Mxy = 0. If Mx and My are known, the bending and twisting moment in any other direction can be graphically obtained by a Mohr cicle. Mx, My may be positive or negative.

Fig.2



By further consideration of all the forces and bending moments acting on the element which are shown below, and the equations of equilibrium, the following basic plate equations are derived :-

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x . \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q \quad \text{........(9)}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = q/D \quad \text{.......(10)}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = -\frac{Q_x}{D} \quad \text{.......(11)}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = -\frac{Q_y}{D} \quad \text{......(12)}$$

where D = flexural rigidity per unit length

$$= \frac{E_c h^3}{12(1-\nu^2)}$$

q = transverse load per unit area.

Ec = Instantaneous Elastic Modulus of Concrete.

h = Thickness of slab.

ν = Poissons ratio (0.15-0.25)

Mx, My, Mxy are moments per unit length expressed in force units.

To find a solution for a given plate it is necessay to integrate (10) successively and determine the constants of integration by satisfying the known boundary conditions.(Two per edge) In this way, an expression is found for "w" in the form of a converging trigonometric series.

By way of illustration for a simply supported square plate a solution is

(R11) $$w = \frac{qa^4}{D}\sum_{m=1}^{\infty}\left(\frac{4}{\pi^5 m^5} + A_m \cosh\frac{m\pi y}{a} + B_m . \frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right)\sinh\frac{m\pi x}{a}$$

where m = 1, 3, 5 ....(13)

Am, Bm are parameters to be determined from the boundary conditions.

Mx, My can similarly be calculated from a converging series.

Design of flat plates through analysis requires that

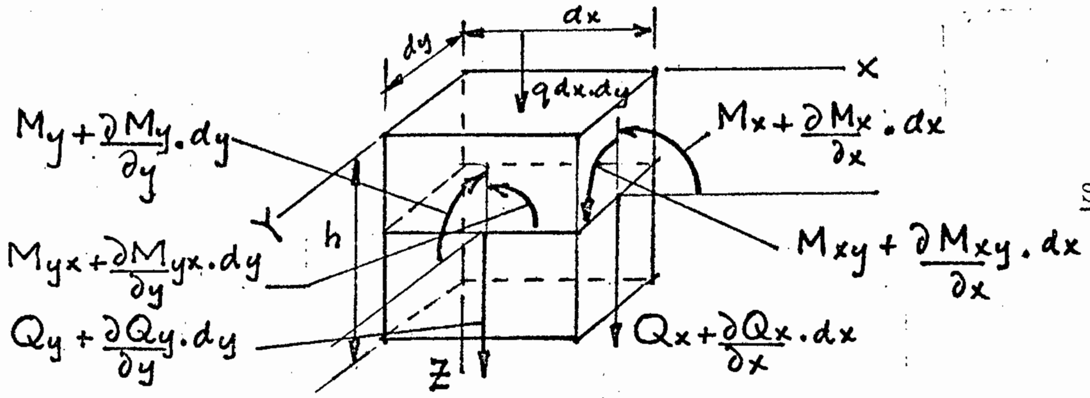


Fig. 3

Slab Element

constants similar to those in the above equation be obtained by the satisfaction at a large number of points of boundary conditions which may change sharply from one panel to the next.

In view of this complexity and the vast amount of computation required to determine the bending moments at selected points in order to proportion the steel reinforcement, the method is not suitable for use in a design office.

An approximate solution to the above with a small margin of error can however be obtained by the method of Finite Differences and this will be discussed in detail at a later stage.

B. ULTIMATE STRENGTH IN BENDING OF FLAT PLATES:

For under reinforced sections, the plastic moment of resistance or ultimate moment per unit width of slab is given by various Codes as follows:-

CP 114 $$m = p\, f_y d^2 \left(1 - 0.75 \frac{p f_y}{u}\right) \quad \ldots\ldots \quad (14)$$

ACI-315-68 $$m = p\, f_y d^2 \left(1 - 0.59 \frac{p f_y}{f_c^1}\right) \quad \ldots\ldots \quad (15)$$

where
- m = ultimate moment per unit width
- p = reinforcement ratio
- f_y = yield stress of the steel
- d = effective depth ofthe steel
- u = concrete cube strength
- f_c^1 = concrete cylinder strength.

Both formulae are identical if $\frac{0.75}{u} = \frac{0.59}{f_c^1}$

or $f_c^1 = 0.79\, u$ which is generally accepted.

(R38) CEB $m = p d^2 \sigma_{au} \left(1 - \frac{\bar{w}}{2}\right)$ where σ_{au} = steel stress

$\sigma_{au} = \sigma_{0.2}(1.2 - 6p)$ $\sigma_{0.2}$ = 0.2 proof stress.

$\bar{w} = p\sigma_{au}/\sigma'_{cyl.}$ σ'_{cyl} = cylinder strength.

For convenience formulae (14) will be used.

Consider a continuous flat plate on columns at "L" centres in the one direction and "l" centres in the direction at right angles and required to carry a total uniformly distributed ultimate load P (See figures 4).

According to yield-line theory the simplified fracture

line /

line pattern consists of parallel positive and negative moment lines in the one direction only which allows the slab to fold.

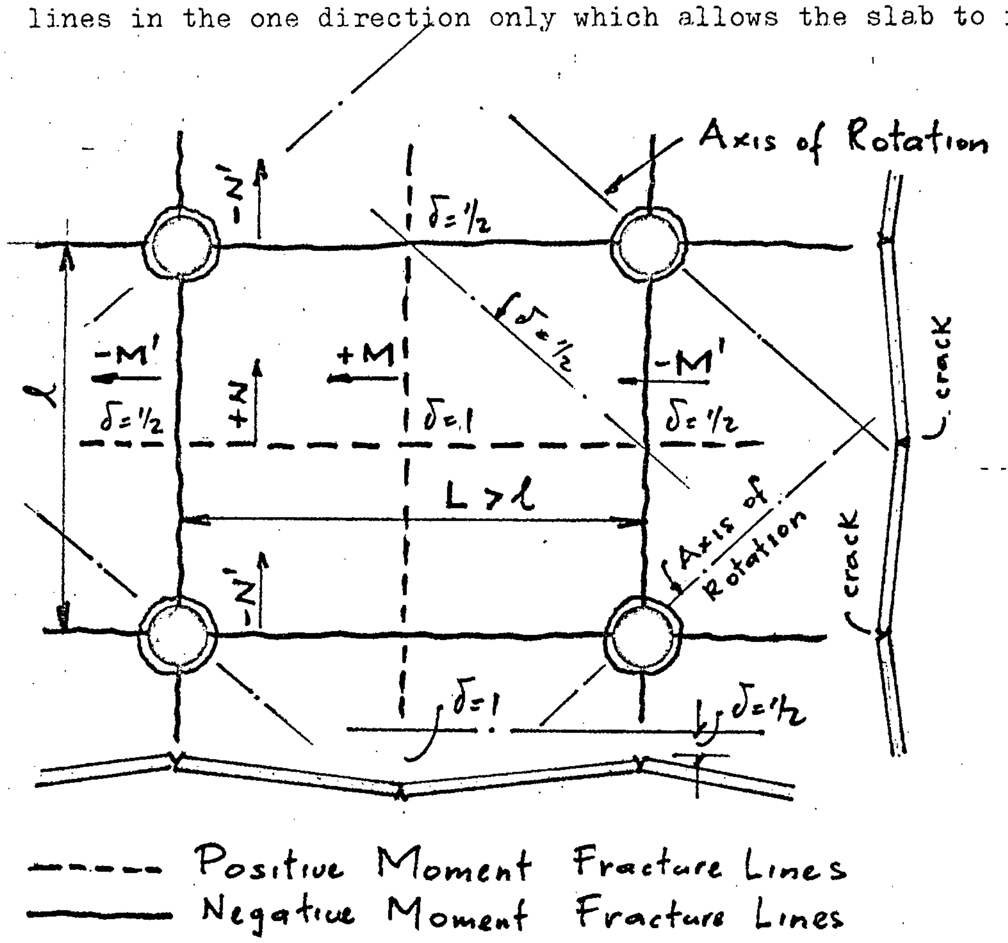


Fig 4 - Fracture Line Pattern and Deflected Shape

If $+M$ = Total positive yield moment of width 1 in direction of L

$-M^1$ = Total negative yield moment of width 1 in direction of L.

then from equilibrium of the full panel L x 1

(R4) $$M + M^1 = Mo = \frac{PL}{8} - \frac{P r_b}{\pi} = P\left(\frac{L}{8} - \frac{r_b}{\pi}\right) \quad \ldots.(16)$$

where Mo = total static moment for the Panel

P = total panel load at failure = = q L 1

$\frac{P r_b}{\pi}$ = reduction in Mo due to column size.

However the slab may fail at lesser values of P by similar fracture lines running in the opposite direction if

$$N + N^1 < P\left(\frac{l}{8} - \frac{r_b}{\pi}\right)$$

where N and N^1 are respectively the total positive and negative yield moments in the direction 1 across the slab width L.

For the panel load at failure to be the same in both directions

$$\frac{M + M^1}{\frac{L}{8} - \frac{r_b}{\pi}} = \frac{N + N^1}{\frac{1}{8} - \frac{r_b}{\pi}} \qquad \ldots\ldots (17)$$

In this way yield lines will form simultaneously in both directions as shown in Figure 4 when the panel load P is reached. Hence $(M + M^1)$ for bending in the one direction and $(N + N^1)$ for bending in the transverse direction can be readily determined. For reasons of economy and optimum behaviour, M and M^1 should be proportioned so that $1.0 \leqslant \frac{M^1}{M} \leqslant 1.5$

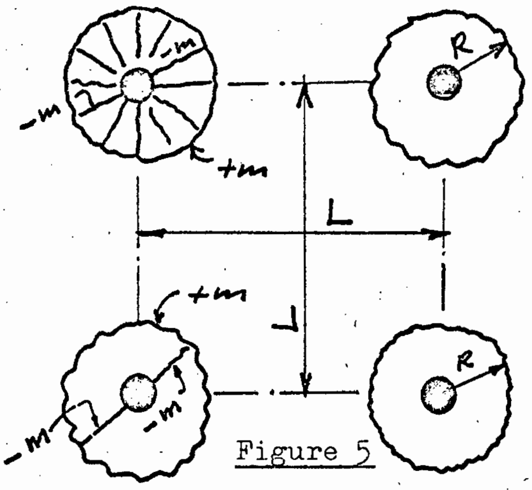


Figure 5

(R35) Due to the column reaction an inverted conical failure mode might develop over each support causing the central rigid portion to deflect equally (Figure 5). In the special case of a square panel of side L with isotropic reinforcement and yield moment "m" per unit length, let R = assumed radius of the failure cone. $m = \frac{M^1}{L}$. Then from the virtual work equation

$$\frac{P}{m} = \frac{4\pi}{1 - \frac{\pi}{3}\frac{R^2}{L^2}} \qquad \ldots\ldots\ldots (18)$$

If $\frac{R}{L} = 0.05 \qquad \frac{P}{m} \approx 4\pi \qquad P \approx 12.6\ m$

Assuming a parallel line mode of collapse (Figure 4)

$$P = 16\ m \quad \text{for } r_b = 0$$

$$\text{and } P \approx 18\ m \quad \text{for } \frac{r_b}{L} = 0.05$$

Hence it will be necessary to increase the value of "m" near the column to $|m| = \frac{18}{12.6} \times m = 1.44\ m$ to prevent collapse by the conical fracture pattern at a lesser load than that determined from the equation (16).

A similar precaution is required for rectangular slabs and is simply achieved by concentrating 50% of the total panel negative moment in 25% of the panel width opposite the column i.e.

$$|m| = \frac{0.5\ M^1}{0.25\ L} = \frac{2\ M^1}{L} = 2\ m > 1.44\ m$$

As stated earlier M^1 should be made $>$ M within the range $1.0 \leq \frac{M^1}{M} \leq 1.5$. This concentration of steel reinforcement directly over the column has the additional benefit of raising the punching shear strength and also reducing the deflections of the slab. (See Elastic-Plastic behaviour).

(R38) Reference is made here to tests on an experimental model made in America at the University of Illinois in 1959. The slab consists of 9 square bays each 1525 x 1525 mm simply supported along the perimeter by either shallow or rigid beams on columns and by 4 central columns. The slab is 45 mm thick. Details of the Model and of the cracks formed on reaching ultimate load are shown in Figures A.F2, A.F3, A.F4 in the Appendix

Two idealized fracture patterns were considered and the ultimate load calculated for each. (Figs. A.F5, A.F6) The experimental failure load of 1760 kg/cm^2 was found to be the exact mean of the two predicted failure loads. In addition the steel stresses were found to reach the yield values over the entire length of the yield line.

This and other tests confirm the close agreement of the predicted values of ultimate load using yield-line theory with the experimental values.

$$\text{Average } \frac{P \text{ test}}{P \text{ calc.}} = 1.2.$$

(C) PUNCHING SHEAR STRENGTH OF SLABS

Most Codes presently determine the allowable or ultimate punching shear at the slab-column junction by considering the shear stress on a vertical section around the periphery as the criterion, although failure is actually in diagonal tension. Depending upon the Code, the vertical section varies from the edge of the column or loaded area to a distance away equal to the effective depth of the slab. For example :-

A.S. No. Ca 2 - 1958 (Australian Standard)

$$v = \frac{Vs}{b_2 d} \leq 0.022\,Fc^1 + 22 \text{ or } 100 \text{ p.s.i. maximum} \quad \ldots\ldots(19)$$

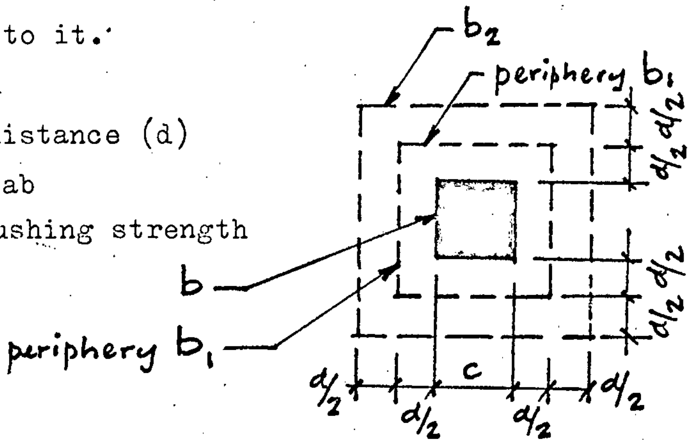
when at least 50% of the total negative steel in the Column strip pass through the periphery b_2 taken at a distance (d) from the column face and parallel to it.

Vs = allowable shear load

b_2 = Total periphery at distance (d)

d = effective depth of slab

Fc^1 = concrete cylinder crushing strength



A.C.I. - 1968

$$v_u = \frac{Vu}{b_1 d} \leq 4.0\sqrt{Fc^1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

b_1 = effective periphery at a distance (d/2) from the edge of the loaded area.

OR

$$v_u = \frac{Vu}{b\,d} = 4\left(\frac{d}{r} + 1\right)\sqrt{Fc^1} \quad \text{(Hyperbolic fitting Equation)} \ldots.(21)$$

b = periphery at edge of loaded area r = side of square column

C.P. 114 - 1967

$$v = \frac{Vs}{b_1 d} = \frac{U_w}{30}$$

The best available analysis of punching shear strength of reinforced concrete slabs appears to be that by Moe for $\frac{r}{d} < 3$

$$v_u = \frac{Vu}{bd} = \left[15(1-0.075\,{}^{r}/_{d})-5.25\,\frac{Vu}{Vflex}\right]\sqrt{Fc^1}$$

$$= (9.75-1.125\,{}^{r}/_{d})\sqrt{Fc^1} \text{ when } \frac{Vu}{Vflex} = 1 \quad \ldots\ldots\ldots(22)$$

Vu = ultimate Shear Load

b = Column perimeter or periphery of loaded area

d = Average effective depth of reinforcement

Fc^1 = Concrete cylinder compressive strength

r = Length of side of square column of equivalent area.

$Vflex$ = Ultimate flexural capacity using Yield Line

For $\frac{r}{d} > 3$ and for specimens with shear reinforcement the experimental formula deduced by Tasker and Wyatt gives excellent results :-

$$v_u = \frac{Vu}{bd} = \left(2.5+\frac{10}{\left[{}^{r}/_{d}\right]+1}\right)\sqrt{Fc^1} \quad \ldots\ldots\ldots\ldots(23)$$

If shear reinforcement is used r = base of the outermost reinforcement. The effect of the shear reinforcement is not to support all or part of the load but to transmit the entire load

to the /

to the surrounding concrete so that failure can only occur outside the shear cage.
Formula (23) will also give good results for $1 < \frac{r}{d} < 7$
The authors recommend that the F.S. for shear should exceed F.S. for flexure by 0.2.

From the results of shear tests on flat slabs carried out at the University of Illinois in 1952 and by the Portland Cement Association in 1954, Whitney has produced a straight line Empirical formula for estimating the ultimate shear strength. Approximately 40 slabs were tested 6 ft. square x 6 in. thick with steel ratios from 0.0055 to 0.037 and with Fc^1 from 2,000 to 5,000 p.s.i. The slabs were loaded through the column stub and supported around their perimeter. All the important variables which greatly influence shear strength, were taken into account e.g. steel and concrete strength, size and spacing of bars, position of applied loading, depth to span ratio and column size. The slabs were reinforced in two directions in which maximum shear is accompanied by maximum bending moment. Three main types of failure were distinguished:-

1. With lightly reinforced slabs i.e. $Mu/_{d^2} \sqrt{d/_{ls}} < 150$ the full flexural strength was reached using yield line theory. After yielding excessive cracking of the concrete occurred due to elongation of the steel, reducing the shear strength until the column punched through.

2. With heavier reinforcement, bars near the column yielded sufficiently to cause cracking leading to punching, before the bars outside the zone of rupture had yielded.

3. In the over-reinforced slabs, the compression zone around the column was destroyed before any steel had yielded causing sudden punching. The author considers that load failure due to too close spacing also contributed. For $Mu/_{d^2} \sqrt{d/_{ls}} = 500$

$$\frac{P\ Test}{P\ flex} = 0.60$$

Most consistent results were obtained by calculating the shear stress at a distance d/2 from the column face and using this as the criterion.

i.e. $$v_u = \frac{P\ ult}{4\ d\ (r+d)} \leq 100 + 0.75\ \frac{Mu}{d^2} \sqrt{d/_{ls}} \quad \ldots\ldots(24)$$

Where r = length of the side of the column
d = effective depth of the steel
ls = "shear span"

Mu /

Mu = ultimate R.M. per in. width within the base of the pyramid of rupture ....(25)

$$= pd^2 fy \left[1 - 0.59 \frac{pfy}{fc^1}\right] \quad \text{when under-reinforced}$$

$$= \frac{d^2 fc^1}{3} \quad \text{when over-reinforced}$$

To prevent bond failure, the bars should be fully anchored beyond the point where they intersect the pyramid of rupture. If the bars are uniformly spaced they should not be closer than

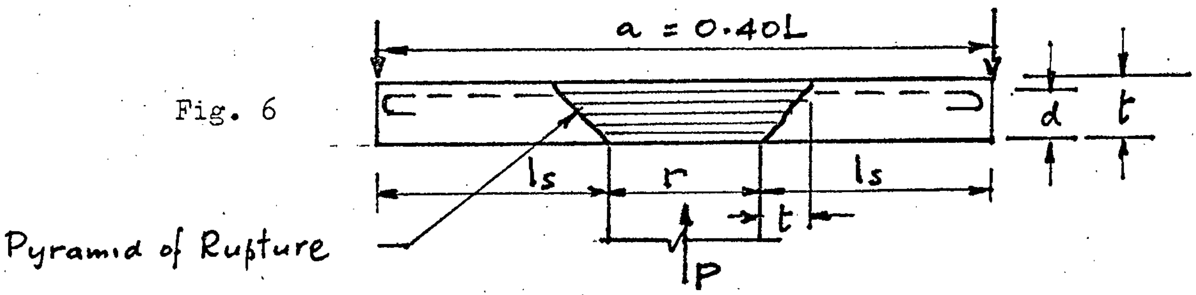
$$\frac{18,000}{fc^1} \quad x \quad \text{bar diameter}$$

Whitney suggests that as in most cases of uniformly spaced bars the shear capacity is less than the flexural capacity, the best procedure would be to first calculate the total amount of reinforcement required for flexure and provide enough steel through the pyramid of rupture to provide the same shear strength using equation (24). The balance of the total steel should then be distributed outside this zone.

The above formula may be used for flat plates by assuming ls = distance from the face of the column to the line of inflexion. As some of the load is distributed inside the lines of inflexion, ls may be reduced slightly.

Assuming a square plate with points of contraflexure at 0.22L, then $a \leqq 0.44L = 0.40L$ say.

Fig. 6



The tests show that shear failure only takes place after local yield of the tensile steel, provided it is less than that required for balanced design at ultimate flexural capacity. Hence prevention of yield will not only prevent shear failure but also unserviceability through excessive cracking. The following load at which local yielding occurs at the columns can then be taken as the lower bound of the ultimate shear strength. (See Elastic-Plastic Design)

For rectangular slab :-

If r_b = effective column radius

$$P2 = \frac{-4\pi \, Mpy}{\text{Log}_e \frac{rb}{Lx} + 2.310 - \frac{Ly}{Lx}} \quad ....(26)$$

where /

where M_{py} = yield moment per unit width in long direction

L_y = Long span

L_x = Short span

ELSTNER & HOGNESTAD (1956)

$$v_u = \frac{V_u}{7/8\, bd} = \left[333 + 0.046\, F_c^1\right] \text{ in p.s.i.} \quad \ldots(27)$$

The punching shear strength of plates is seldom a limiting criterion however. For optimum slab performance, it will later be shown that the slab depth is chosen so that the modulus of rupture is not exceeded at mid-span. The resulting slab depth normally provides adequate shear strength.

(R29) Should it be necessary however, the shear strength can be increased by placing a structural steel shear-head over the column and within the slab designed to take at least 75% of the full punching shear. The length of the arms are selected so that the ultimate shear stress at the critical section shown which connects the extremeties of the shear head arms, is not exceeded at ultimate load. Figure (7) shows a typical detail. Tests on other types of shear reinforcement show that the function of the shear reinforcement is to distribute the column load to a sufficiently large area of the adjoining slab so that shear failure can only occur outside of it.

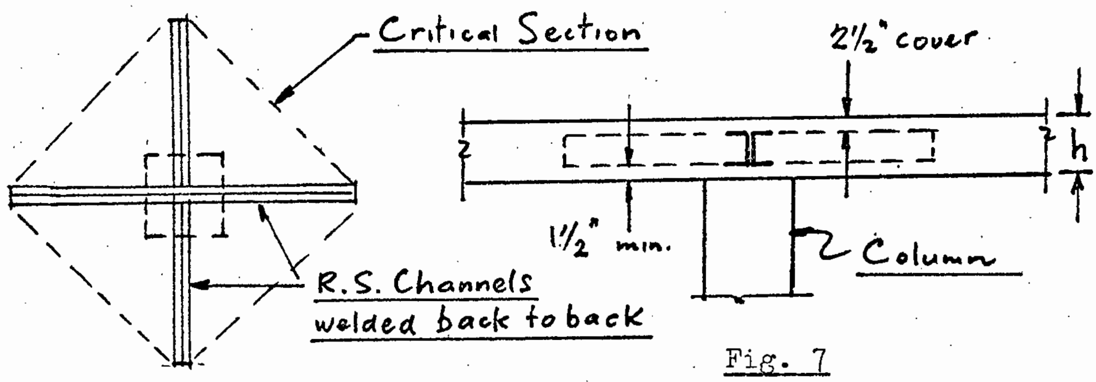


Fig. 7

Various other authors also report excellent results by using steel grillages over the columns to transmit the column loads safely into the slab.

(D) DEFLECTIONS /

(D) DEFLECTIONS OF FLAT PLATES

The methods most commonly used for the analysis of Flat Plates are based upon either the criterion of "allowable stress" or the criterion of "ultimate load". This treatment is therefore the same as the conventional treatment of structural members.

There is increasing evidence however, that deflection should be considered as an additional criterion, either because it may be associated with large strains and cracking of the concrete, or because it may affect other parts of the structure, principally brittle partitions. The principal difficulties to using this criterion are however :-

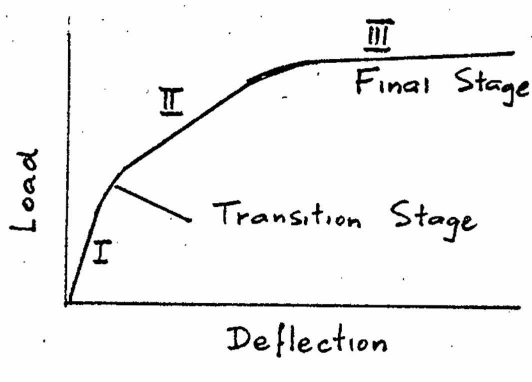
1. A suitable method of calculation of the deflection
2. The rational choice of the limits of deflection.

Traditional design by "allowable stresses" of Timber structures introduced deflection indirectly as a criterion by setting a limit of span/360. This was supposed to prevent visible cracking of plaster surfaces and badly sagging floors. Youngs Modulus for Timber under sustained loading was taken at $\frac{1}{3}$ that for short-term loading. This treatment curiously resembles the present treatment of concrete members.

In the case of concrete structures however, which consist of beams and slabs and panel walls between floor beams, inter-action or composite action reduces stresses and deflections to a fraction
(R37) of the calculated values. In contrast, flat plate structures behave fairly closely to the design assumptions and although satisfactory in all respects can have deflections less than span/360 which cause severe cracking of internal partitions.

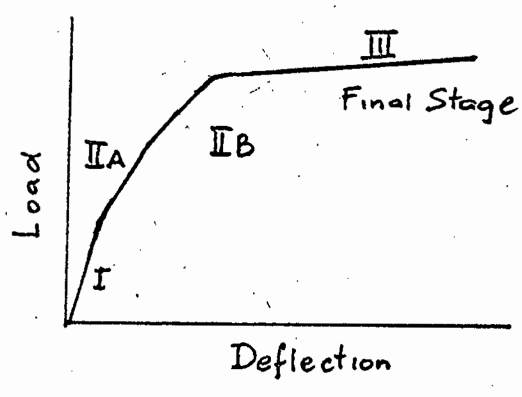
Short-Term Deflections of Slabs

The basic form of the load-deflection relationship is shown in Fig (8)



Beams and One-way slabs

Fig. 8(a)



Flat Plates

Fig. 8(b)

In /

In Stage I which is linear, the concrete section at mid-span remains uncracked and the deflection is governed by the usual moment-curvature relation

$$\frac{d^2y}{dx^2} = \frac{M}{EcIg} \quad \text{..............(28)}$$

where $\frac{d^2y}{dx^2}$ is the usual approximation for the curvature

Ec = short-term modulus for concrete

Ig = moment of inertia of the gross section neglecting the steel

In Stage II the concrete has cracked at all critical sections as well as over large portions of its length, wherever M $>$ Mcr (cracking moment). The deflection is governed by the following relation at cracked sections, which neglects the stiffness of the concrete in tension between the cracks

$$\frac{d^2y}{dx^2} = \frac{M}{Es\ As\ d^2(1-k)(1-k/_3)} \quad \text{.........(29)}$$

where Es = modulus of elasticity of the steel

As = area of tensile reinforcement

k = parameter defining the depth of the neutral axis and is given by

$$k = \sqrt{pn\ (2+pn)} - pn$$

For lightly reinforced sections $k \approx 3/8$

$$\text{and} \quad \frac{d^2y}{dx^2} = \frac{M}{0.6\ Es\ Asd^2} = \frac{M}{Es\ I^1} \quad \text{.......(30)}$$

where $I^1 = 0.6\ Asd^2$

If the average M over the beam is high e.g. simply supported beam, equation (29) will apply over a large portion which can be taken as the whole beam with no serious error. Hence Stage II will be nearly linear and the transition between I and II will be brief. In other cases such as a continuous beam, equations (28) and (29) must be applied to the appropriate zones of the beam and the transition will be gradual. Stage III is characterized by yield of the reinforcement. However if high shear combined with moment at the critical section has caused bond failure, the stiffness of that section may be reduced to one-half that given by equation (29).

Centres of panels of flat plates show the same basic pattern of deflection under load (Fig. 8(b)) but the transition stage is usually so long that the deformations are quite large before the second stage is reached. Experiments show that cracking starts in/

in the neighbourhood of the column head and is confined to a fairly small roughly circular zone, having no noticeable effect on the overall slab stiffness, especially as the reinforcement ratio in this zone is high. Only when the modulus of rupture is surpassed at the mid-span section does the slope of the curve change. Cracking then spreads widely over a zone which is the most lightly reinforced in the whole slab causing a large overall reduction in stiffness. Calculation of the average stiffness at any cross-section will be difficult however because of the moment variations across the width of the panel causing portions to be cracked or uncracked. This variation is greatest in the negative moment regions. See Fig. (9). Furthermore in the cracked zones, variation in the amount of reinforcement in the column and mid strips respectively will also greatly vary I^1.
To complicate the calculations further in a rectangular plate, each direction will have a different cracking load which will be roughly proportional to the inverse of the square of the span.

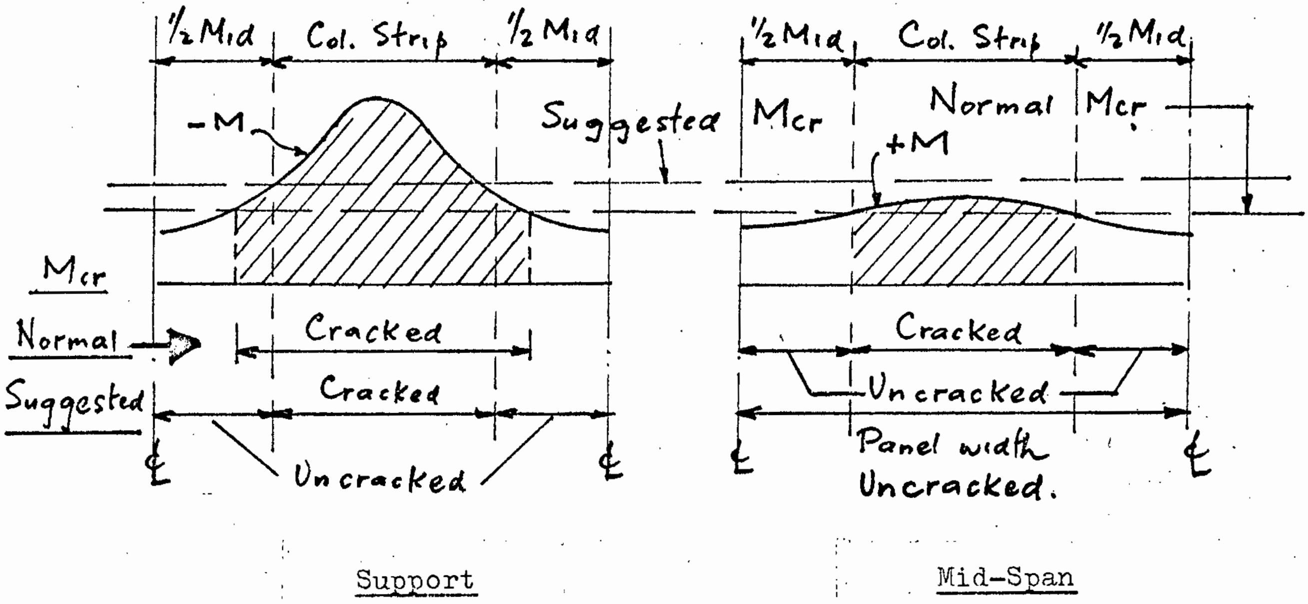


Fig. 9 Cracked and uncracked portions of full panel width

Calculations of deflection will be greatly simplified if the cracking moment (Mcr) of the slab were only to be exceeded within the column strip at the column face and in the longer direction of span. Cracking would then be confined to a small zone around the column of area approximately 10% of the panel area in the case of a square panel. The stiffness Ec Ig of the uncracked section could then be used in all deflection calculations without serious error.

As the use of equations (28) and (29) simultaneously is not practical, the following approximate procedure is suggested

for /

for the estimation of deflections and has been found to yield fairly reliable results :-

1. Treat the plate as an elastic frame firstly in the one direction carrying the full design load and then the other direction. Calculate the deflection in each case as shown below and then add both deflections together.

2. Calculate the load required to crack the panel in the centre.

3. For loads $<$ cracking load, calculate the deflection on the basis of the uncracked section stiffness. (Equation 28).

4. For loads $>$ cracking load calculate the deflection <u>only for the margin of load</u> above the cracking load, using the cracked section stiffness (Equation 29) or table 1 (See below).

5. For ratios of sides $>$ 1.33 the deflection contribution from the short span can be neglected.

To assist in calculations, the cracked section stiffness of under-reinforced sections for various percentages of reinforcement and the short-term Concrete Modulus Ec are given in the following Table :-

Table 1 Stiffness Parameters for Cracked Reinforced Concrete Sections

n	p	$\frac{El}{bd^3} \times 10^{-6}$
5.0	0.005	.112
$E_c=6x10^6$ p.s.i.	0.010	.211
	0.015	.274
	0.020	.339
	0.025	.398
7.5	0.005	.105
$E_c=4x10^6$ p.s.i.	0.010	.182
	0.015	.246
	0.020	.301
	0.025	.348
1.00	0.005	.100
$E_c=3x10^6$ p.s.i.	0.010	.170
	0.015	.225
	0.020	.272
	0.025	.313

D.1. Alternative Methods of Deflection Prediction Suggested by the Writer

(R39) In its report on the deflection of R.C. flexural members, A.C.I. Committee 435 recommends inter-alia the following methods for predicting the short-term deflections :-

1. Branson 1963

When Mmax $>$ Mcr use the average effective stiffness given by

$$I_{eff} = \left[\frac{M_{cr}}{M_{max}}\right]^3 I_g + \left[1 - \frac{M_{cr}}{M_{max}}\right]^3 I_{cr} \quad \ldots(31)$$

If Mmax $<$ Mcr use $I_{eff} = I_g$

If the beam is continuous, use the average of the I eff values for the positive and negative M regions.

$$M_{cr} = \frac{f^1_{cb}\, I_g}{Y_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots(32)$$

where Mcr = cracking moment

f^1cb = modulus of rupture

Ig = second moment of area of the gross section neglecting the steel

Icr = second moment of area of cracked transformed section

Yt = distance of extreme tensile fibre from the N.A.

$$E_c = 33\sqrt{w^3 F_c^1} \text{ p.s.i. for w in per ft.}^3$$

$$= 57{,}600\sqrt{F_c^1} \text{ p.s.i.}$$

2. A.C.I. Code 1963

Use I = Ig when pfy $<$ 500 p.s.i.

I = Icr when pfy $>$ 500 p.s.i.

If the beam is continuous use the average of the values obtained for the positive and negative moment regions.

By applying methods (1) or (2) to determine I eff of the full panel width which is assumed to be constant over the entire panel span, calculate the deflections Δ_x and Δ_y in each direction under the full load and add together for the deflection of the panel centre.

(R31) Reference is also made to an approximate method developed to predict deflections of floor slab systems with or without beams based on "exact" or finite difference solutions for interior rectangular panels and test data from 5 structures.

A Physical analog is proposed as a substitute for the frame/

frame consisting of beam and plate elements delimited by the idealized lines of contraflexure at distances of 0.2 L and 0.2 S from the column lines (L = Long span, S = Short span). The beam elements have widths of 0.4 L and 0.4 S respectively supported by columns at S centres and L centres respectively and the plate element is the portion of slab bounded by the beam elements.

The total deflection at the centre of an internal panel would then be the sum of :-

(a) Deflection Δa at the centre of one of the longer beams in respect to its supporting columns.

(b) Deflection Δb at the edge of the beam in respect to its centre.

(c) The deflection Δc of the plate element with respect to the edge of the beam.

Δa is calculated by the elastic analysis of the beam and Column frame (Ersatz Frame). $\Delta b + \Delta c$ is approximately equal to the mid-panel deflection of a rigidly clamped plate of the same size and is calculated by approximate methods.

Different boundary conditions are taken into account by calculating the rotation θ of the panel at the boundary from a full panel elastic analysis, and by adjusting for this rotation. This method takes into account the restraints offered by the column stiffnesses as

$$\theta \text{ is then} = \frac{\text{Mcol.}}{\text{Col. stiffness}}$$

Measured load-deflection curves are compared with curves obtained from the approximate method on the basis of both cracked and uncracked sections. It was observed that short time deflections under working load are better predicted on the basis of uncracked sections since a large portion of the slab is uncracked. Furthermore slab construction under working load owes its successful behaviour to the tensile strength of the concrete.

As there is little published evidence available that the methods outlined above give results in reasonable agreement with prototype flat plates, they must be regarded as first approximations only. It appears more practical to restrict deflection by indirect means which will be outlined hereunder.

D.2. INDIRECT CONTROL OF DEFLECTION

As a basis, it may be assumed for practical purposes that gypsum plaster cracks at a strain of 5×10^{-4} and concrete at a strain of 1×10^{-4} irrespective of its strength. The cracking strain of brickwork is believed to lie between these values and will be estimated from tests.

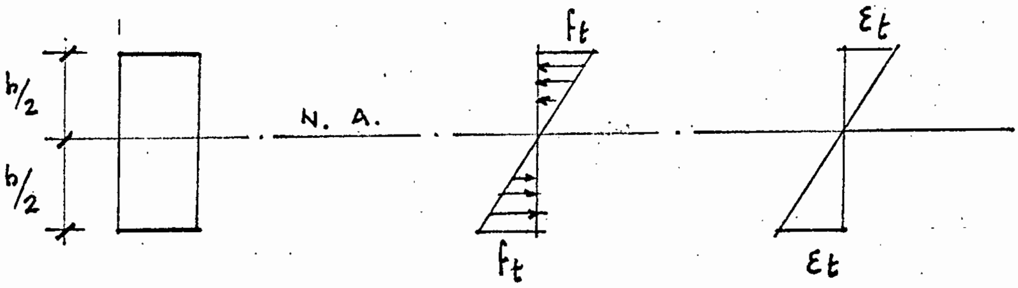
For a uniform elastic beam with N.A. at mid-depth

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \qquad \varepsilon = \frac{f}{E} = \frac{y}{R}$$

$$\frac{1}{R} = \frac{\varepsilon}{y} = \frac{2\,\varepsilon_t}{h} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

where ε_t = extreme fibre strain h = overall depth

(See Fig. 10)



Cross-Section Stress Strain

Fig. 10

Consider the case of a simply supported slab span L with total load W uniformly distributed. (Fig. 11)

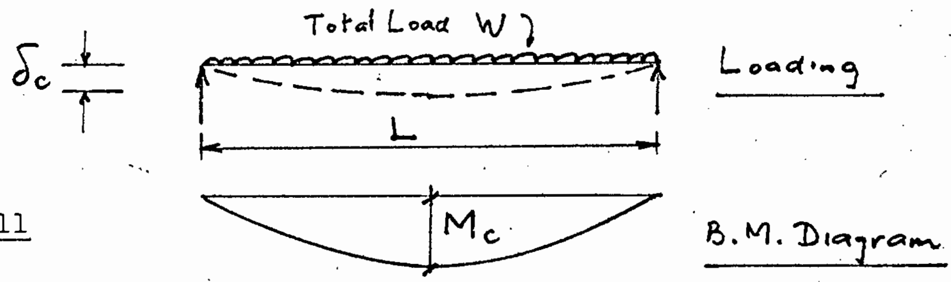


Fig. 11

$$\text{Mid-span deflection} \quad \delta_c = \frac{5}{384}\frac{WL^3}{EI} = \frac{5}{48}\frac{M_c L^2}{EI} \qquad \ldots\ldots\ldots(34)$$

$$\therefore \quad \frac{\delta_c}{L} = \frac{5}{48} \cdot \frac{1}{R_c} \cdot L \qquad \ldots\ldots\ldots\ldots\ldots\ldots(35)$$

If slab is designed for no cracking at mid-span i.e. $M_c < M_{cr}$

then $\varepsilon_t = 1 \times 10^{-4}$ maximum

$$\text{From (33) and (35)} \quad \frac{\delta_c}{L} = \frac{5}{48} \cdot \frac{2\varepsilon_t}{h} \cdot L = \frac{1}{48{,}000} \cdot \frac{L}{h}$$

$$\text{If } \frac{L}{h} = 32 \text{ then } \frac{\delta_c}{L} = \frac{1}{1500} \qquad \ldots\ldots\ldots\ldots\ldots(36)$$

The above deflection δ_c refers only to the immediate elastic deflection under load. If the load is sustained for 5 years or longer, then due to creep and shrinkage which are time dependant, further cracking and additional movements develop which increase the deflection by an estimated 200% of its initial value. Table (2) shows the multipliers F proposed by the A.C.I. for the Long-term deflection in terms of the initial value with a maximum value of 3. It should be noted that by increasing the age or strength of the concrete at the time of loading by curing or by delaying the application of the load, the time dependent deflection will be reduced substantially.

Duration of Loading	Amount of compression reinforcement		
	$A'_s = 0$	$A'_s = 0.5A_s$	$A'_s = A_s$
1 month	1.6	1.4	1.3
3 months	2.0	1.8	1.6
1 year	2.4	2.0	1.8
5 years or more	3.0	2.2	1.8

MULTIPLIER F FOR LONG-TERM DEFLECTION.

Table 2

Masonry partitions are too rigid to follow the floor deflections and so they span from end to end possibly supporting some of the weight of the slabs or walls above. This will cause severe cracking especially at openings. As only the creep deflection of the floors influences the walls, it is necessary to limit this. The American Society of Civil Engineers limits the creep deflection to span/500 for concrete-block masonry without any information to substantiate it. On this basis it would seem desirable to limit still further the creep deflection of slabs carrying brick masonry.

The following criteria are therefore suggested as an indirect means for limiting deflection and cracking :-

C1. Design for no cracking at mid-span

C2. Ratio of $\frac{\text{span}}{\text{slab depth}} \leq 32$ for simple spans

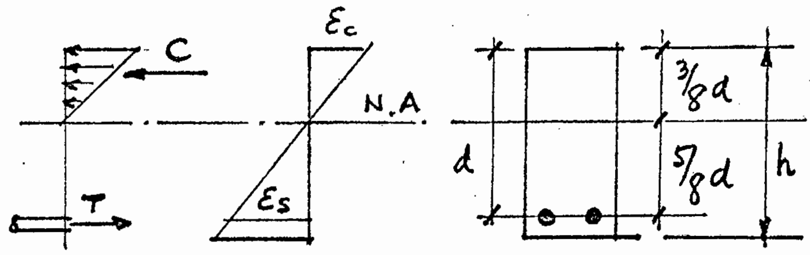
These requirements will result in an initial deflection of span/1500 and a creep deflection twice as much namely span/750 which appears to be acceptable to brick partitions.

Should requirement (1) above not be met the effect on the deflection is illustrated in the following manner :-

In a /

In a reinforced concrete section cracked in flexure, the steel is approximately at 5/8 d from the N.A. and equation (33) becomes $\frac{1}{R} = \frac{8\varepsilon_s}{5d}$ ....(37)

Fig. 12



Stress Strain Section

where ε_s = strain in tensile reinforcement

d = effective depth

If s = modulus of the reinforcement = 30×10^6 lb/in^2

and fs = tensile stress in steel at mid-span

= 30,000 lb/in^2 say

then $\varepsilon_s = \frac{30,000}{30,000,000} = 1 \times 10^{-3}$

From (35) and (37)

$$\frac{\delta_c}{L} = \frac{5}{48} \cdot \frac{8\varepsilon_s}{5d} \cdot L = \frac{1}{6000} \cdot \frac{L}{d} \quad \text{..................(38)}$$

If $\frac{L}{h} = 32$ $\frac{L}{d} = \frac{32}{.88} = 36$ say as $\frac{d}{h} \approx 0.88$ for slabs.

$$\therefore \frac{\delta_c}{L} = 36/6000 = 1/165 \quad \text{for } \underline{f_s = 30,000} \text{ p.s.i}$$

$$= 1/275 \quad \text{for } \underline{f_s = 18\,000}$$

Hence mild steel stressed to 18,000 p.s.i. would be required and the deflection would still be more than 5 times the deflection of $\frac{\text{span}}{1500}$ achieved by meeting requirement (Cl.). Although the depth of slab " h " must be increased to do so, a reduced quantity of high tensile reinforcement can then be used to offset the cost of the extra concrete. The benefits of a crack free slab and partition walls fully justify the small additional expense.

E. METHODS /

E. METHODS OF DESIGN OF FLAT PLATES

E.1. ELASTIC METHOD

Design by the above method may be carried out either in accordance with the American Concrete Institute (A.C.I.) Building Code 318-63 sections 2101 - 2104, or the British Standard Code of Practice C.P. 114 Clauses 325 - 332. Both methods are essentially the same with small variations which will be described. It is expected that the European Codes are also similar in their methods.

The following assumptions are made and all sections are proportioned for the moments and shears so obtained.

1. The structure may be considered to be divided longitudinally and transversely into frames consisting of a row of columns and strips of supported slabs with a width equal to the distance between the centre lines of the panels on either side of the columns.

2. Each frame may be analysed in its entirety or each floor or roof may be analysed separately with the columns above or below fixed at their extremeties. Any suitable method of analysis may be used e.g. Moment Distribution.

3. The spans used in the analysis should be the distances between centres of supports. For the purpose of determining the relative stiffness of the members the moment of inertia of any section of slab or column shall be taken as the gross section of concrete alone. Any variation in the moment of inertia along the axes of the slabs and columns should be taken into account. (In flat plates there is no variation). Joints between columns and slab are to be considered rigid (infinite moment of inertia).

4. The maximum bending moments near mid-span of a panel and at the centre line of the supports should be calculated for the following arrangements of the full imposed loads :-

(i) Alternate spans loaded and all other spans unloaded
(ii) Any two adjacent spans loaded and the other unloaded

The above requirements are from C.P. 114. A.C.I. 318-63 assumes that the maximum bending moments at the critical sections occur under $\frac{3}{4}$ full live loads, as conditions (i) and (ii) cannot occur simultaneously and hence some redistribution of moment is possible. However the design moments taken must not be less than those occuring with full live load on all panels.

5. The slab should be designed for the bending moments so calculated at any section except that the critical section for negative moment is assumed to be the same as the critical section for shear (which is d/2 from the face of the column.) In all cases the numerical sum of the maximum positive bending moments and the average of the negative bending moments (at the critical sections assumed) used in the design of any one span of the slab should be not less than :-

$$\text{CP 114} \qquad Mo = 0.10\ WL \left(1 - \frac{2C}{3L}\right)^2 \qquad \text{..........(39)}$$

$$\text{A.C.I. 318-63} \qquad Mo = 0.10\ WLF \left(1 - \frac{2C}{3L}\right)^2 \qquad \text{..........(40)}$$

where

Mo = numerical sum of positive and negative moments

W = Total load on panel

L = Span length centre to centre of supports

C = effective support size (or average)

= side of equivalent square column

$F = 1.15 - \frac{C}{L} \nless 1.0$

5. (a) A.C.I. 318-63 requirement for Mo above applies when the steel is to be proportioned for each section by the Load factor method. If the elastic method is used the requirement will be $Mo = 0.09\ WLF \left(1-\frac{2C}{3L}\right)^2$ ....(41)

The value of Mo represents the total static moment for which each direction of the panel must be designed. The use of the 0.09 factor instead of 0.125 is claimed to be justified by the experience of many years service with flat slabs having standard capitals and accounts for the so-called plate effects. The factor "F" which is greater than one, is provided to protect flat plates using smaller columns. For example if $\frac{C}{L} = 0.05$ $F = 1.10$

$$\text{and } Mo = 0.09\ WLF \left(1- \frac{2C}{3L}\right)^2$$

$$\doteqdot 0.10\ WL \left(1 - \frac{2C}{3L}\right)^2$$

which is back to the formula from C.P. 114

6. Each panel shall be considered as consisting of strips in each direction as follows:-

(i) A middle strip one-half panel in width and symmetrical about the panel centre-line

(ii) A column strip consisting of the two adjacent quarter panels one on either side of the column centre line.

The/

The bending moments which have been calculated for the full panel width at the critical sections should be divided between the column and mid-strips as follows:-

CP.114 Table 21	Column Strip		Middle Strip	
* A.C.I. 318-63 Table 2103 (c)	CP114	ACI	CP114	ACI
Negative moments at internal support	75	76	25	24
Positive "	55	60	45	40
Negative " at external support	75	80	25	20

* These percentages may be varied by 10% provided the total is the same.

Distribution of bending moments between Column and Mid Strips in percent of the total moment at the critical sections

7. The shear on vertical sections following a periphery b_1 at a distance d/2 from the face of the column shall be computed and limited as follows :-

$$\text{CP 114} : v_s = \frac{Vs}{b_1 d} \leqq \frac{Uw}{50} + 40 \leqq 130 \text{ p.s.i} \quad \ldots(42)$$

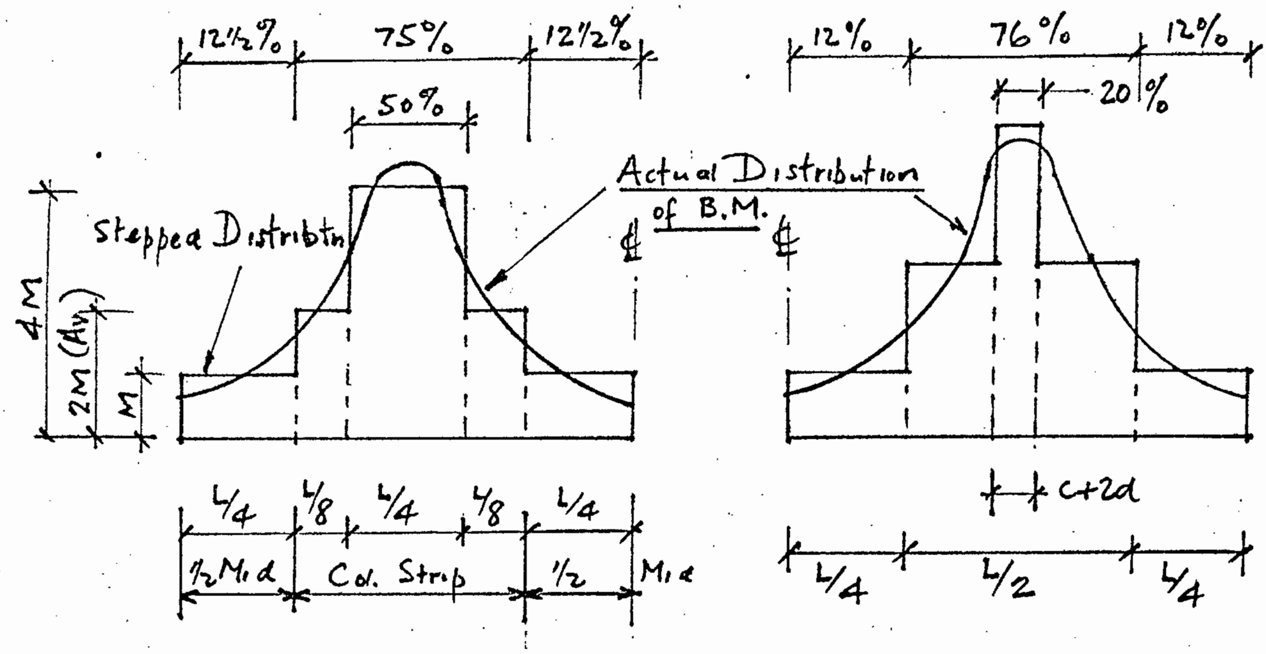
$$\text{ACI 318-63} : v_u = \frac{Vu}{b_1 d} \leqq 4\sqrt{Fc^1} \text{ p.s.i.} \quad \ldots\ldots\ldots\ldots(43)$$

Uw = cube strength $\quad$ Fc^1 = cylinder strength

8. Bars should be spaced uniformly across each panel strip in both directions except

CP 114 - 50% of the total panel negative moment steel should be uniformly spaced across the middle 50% of the column strip

ACI 318-63 At least 25% of the total negative reinforcement in the column strip (± 20% of total steel in panel) should cross a periphery located at a distance d from the column. Spacing of bars = 2h maximum



CP 114 ACI 318-63

Fig. 13 Stepped Distribution of Reinforcement at Critical Negative Moment Section

c = column width d = slab effective depth

9. Total thickness of the slab shall not be less than

CP 114 : 5", $\frac{L}{32}$ for end panels, $\frac{L}{36}$ for internal panels

ACI 318-63 : 5", $\frac{L}{36}$ or $0.028L\left(1-\frac{2c}{3L}\right)\sqrt{\frac{2{,}000w^{1}}{Fc^{1}}} + 1\frac{1}{2}$"
(Elastic)

A.C.I. (Load Factor design) Table 2101 (e)

fy	minimum slab thickness	
40,000	L/36	or 5"
50,000	L/33	
60,000	L/30	

10. Maximum Bending Moments in Columns shall be determined from the condition of full live load on one adjacent panel and the other adjacent panel unloaded.

11. In the case of flat plates designed by the Elastic Method which fall within the limitations required by the Empirical Method (see section E2) the A.C.I. code permits the resulting analytical moments to be reduced by a constant proportion so that the value of Mo is the same as that permitted in the Empirical Method. If this is done, it amounts to a redistribution of the moments obtained by the Empirical Method, from the critical negative moment section to the critical positive moment section, necessitating slightly more reinforcement.

Under these circumstances it would be more feasible to /

to design by the Empirical Method in all cases where the limitations are satisfied. The value of this dispensation is therefore queried. No similar dispensation is made in CP.114

E.2. EMPIRICAL METHOD

Design Bending Moments are given at the critical sections which are justified by the behaviour and ultimate capacity of similar tested structures. They apply one when the following limitations are satisfied :-

1. The construction shall consist of at least 3 continuous panels in each direction.

2. The ratio of length to width of any panel is not greater than 1.33

3. The grid pattern shall be approximately rectangular. Successive span lengths in any direction shall not differ by more than in CP.114 - 10% and in A.C.I. - 20% of the longer span. End spans to be equal or less than interior spans. In the A.C.I., columns may be also offset a maximum of 10% of the span in direction of the offset.

4. Critical sections for the bending moments are given as follows :-

   (i) Positive moment along the centre lines of the panel

   (ii) Negative moment along the line adjacent and parallel to the column centre line and a distance of d/2 from the column face.

5. Numerical sums of positive and negative moments (Mo) is given by

$$\text{CP.114}: \quad M_o = 0.10\, WL \left(1 - \frac{2c}{3L}\right)^2$$

$$\text{ACI}: \quad M_o = 0.10\, WLF \left(1 - \frac{2c}{3L}\right)^2 \text{ for Load-factor design of sections}$$

$$= 0.09\, WLF \left(1 - \frac{2c}{3L}\right)^2 \text{ for Elastic design of sections}$$

6. Distribution of B.M. between Column and Mid-strip as percentages of Mo are given below in plan form by ACI - Section 2104

	Interior Support	Interior Centre	1st Interior Support	Exterior Centre	Exterior Support
Half Col. Strip	-23	+11	-25 (-23)	+14	-20
Middle Strip	-16	+16	-18 (-16)	+20	-10
Column Strip	-46	+22	-50 (-46)	+28	-40

← Direction of Moments →

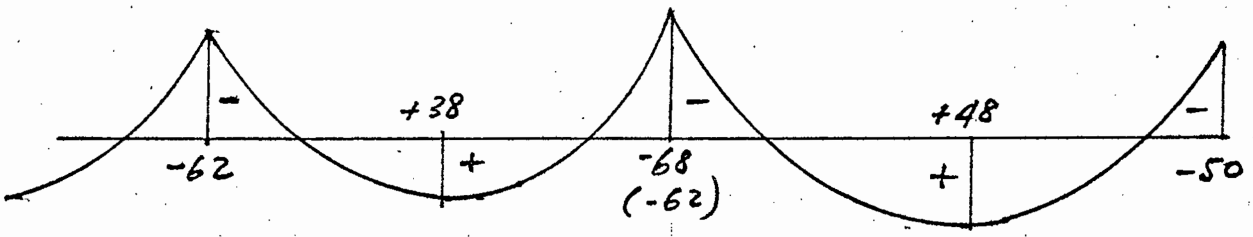


Fig. 14 Design Moment Diagram

Distribution given by CP 114 is identical to the above except for the percentages given in brackets.

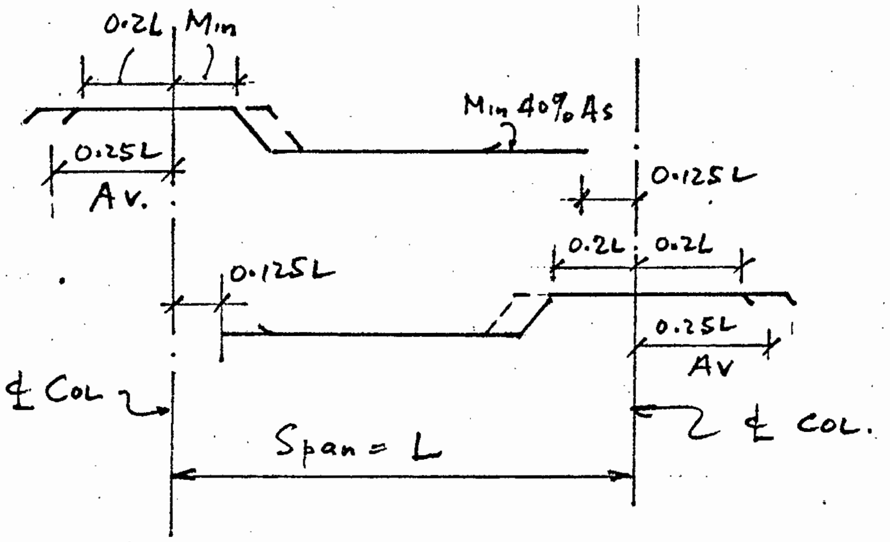


Fig. 15 Arrangement of Bars in Column and Mid Strips

7. B. M. in Columns

CP 114 : Internal Columns M_c = 50% of Negative M in Column Strip

= 23% Mo

External Columns M_c = 90% of Negative M in Column Strip

= 37% Mo

ACI 318-63:Columns to be proportioned for B.M. developed by inequal loading on panels or uneven column spacing so that

$$Mc = \frac{WL_1 - W_D L_2}{f} \qquad \ldots\ldots\ldots\ldots\ldots (44)$$

f = 30 for External Columns

= 40 for Internal Columns

In both Codes, Mc is to be divided between the upper and lower column in proportion to their stiffnesses.

8. Minimum thicknesses of slab are specified in the same way as the Elastic Method.

9. Shear on the critical sections is limited in the same way as in the Elastic Method.

EE. EXAMPLES OF PLATE DESIGN

EE.1. ELASTIC METHOD C.P 114

EE.2. EMPIRICAL METHOD A.C.I 318-63.

EE.1. ELASTIC METHOD (Fig 16)

Panel size $L_1 = 8000$ $L_2 = 6000$ Average $L = 7000$

Slab thickness in end bay $\geq \frac{L}{32} \geq \frac{7000}{32} = 230$

Dead Load - Slab $= 2400 \times ,230 = 552\ Kg/m^2$
$= 5,4\ KN/m^2$

Finish ,5 "

Live Load (150 p.s.f) $= 7,2$ "

Total Load $= 13,1\ KN/m^2$

Check depth for Shear

At internal Col. $P = 13,1 \times 6 \times 8 \times 1,15 \times 1,15 = 825\ KN.$

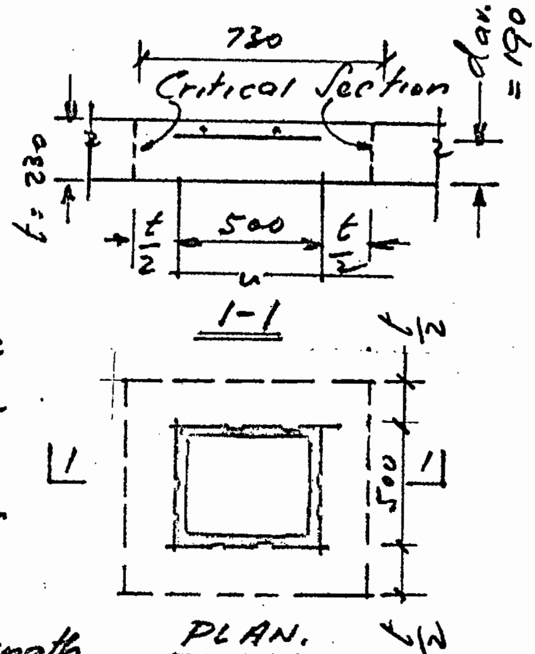
At ground floor Col Load $= 3 \times 825 = 2475\ KN.$

Choose Col 500×500 Direct stress $c = \frac{2475 \times 1000}{500 \times 500}$

$= 10 N/mm^2$ (1500 psi)

Design Col. strength $= \frac{10}{0,3} = 33\ MPa$

Percentage of symmetrical reinfmt. will be determined from BM in Column.



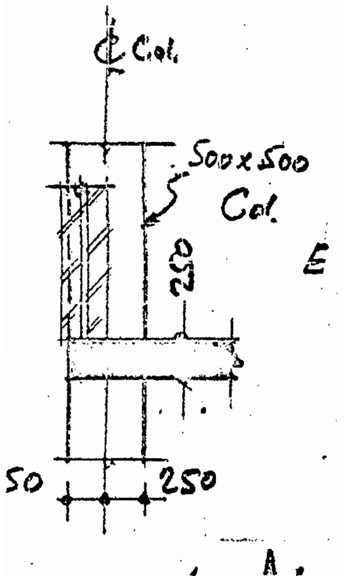
Allowable stress at critical section at dist $\frac{t}{2}$ from Col. face

$= \frac{u_w}{30} + ,275 \leq 0,9\ MPa$

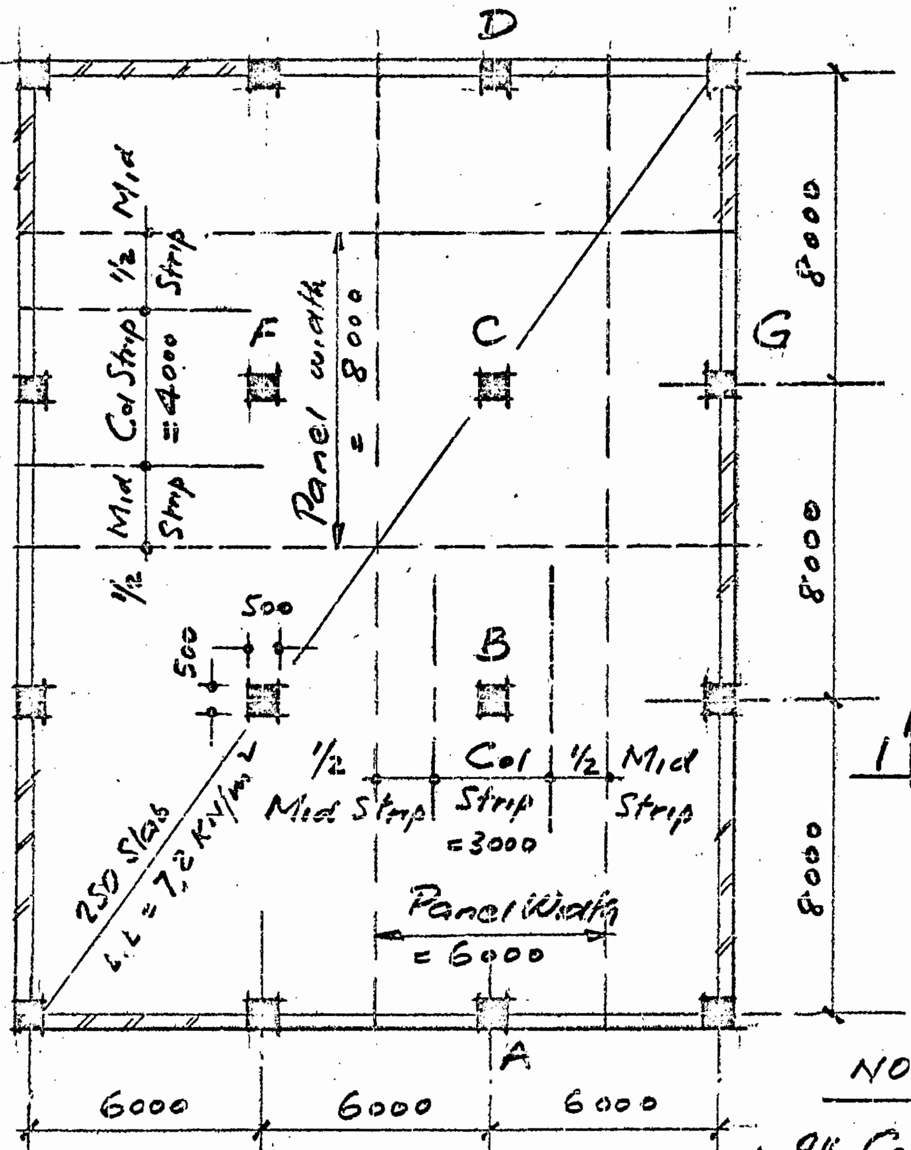
$= 1,1\ MPa$ for $u_w = 25\ MPa$

Shear $v = \frac{825 \times 1000}{4 \times 730 \times 190} = 1,48\ MPa$

As $v >$ permissible shear stress one of the following alternatives are necessary if the slab strength



Detail at A



PLAN - Typical Floor

NOTE

1. All Cols 500 x 500
2. Slab strength = 25 MPa (Cube)

FLAT PLATE DESIGN

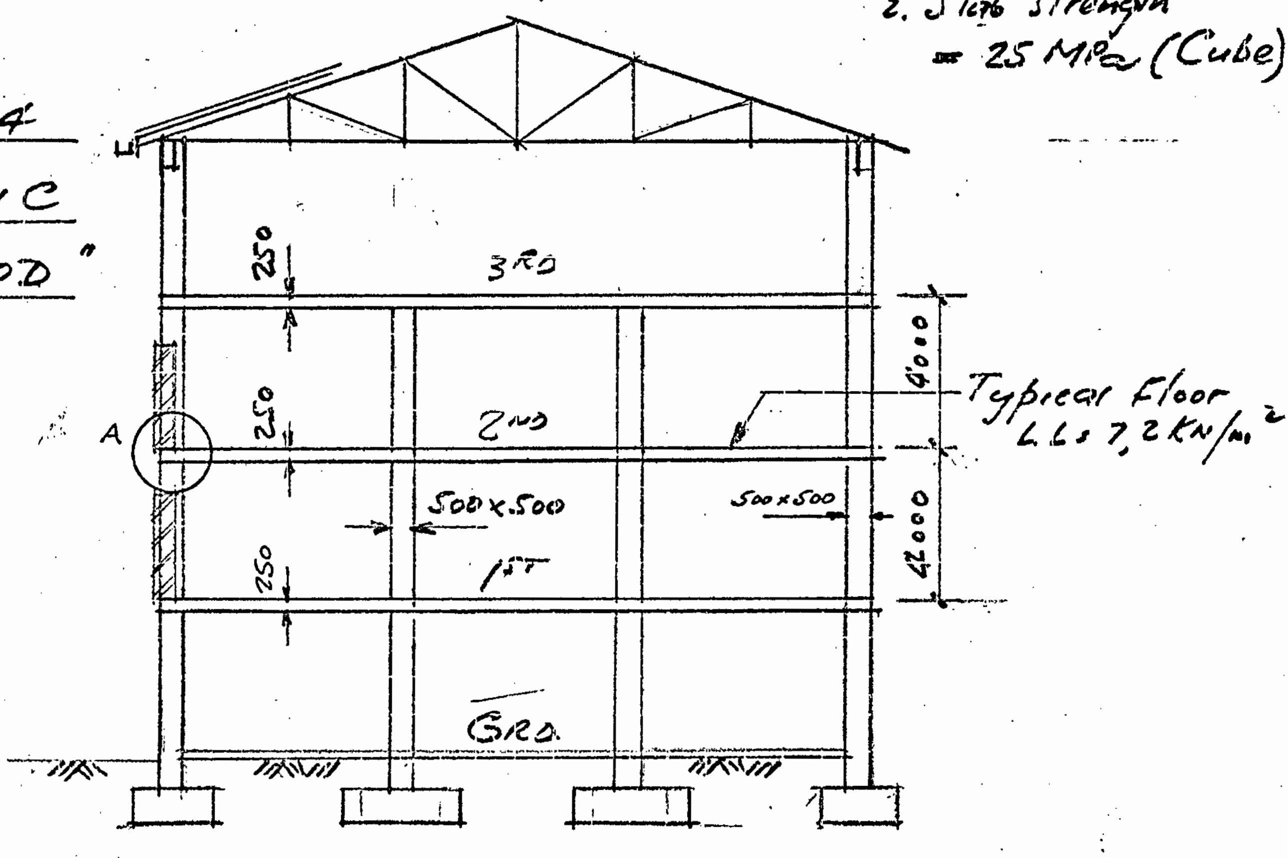
by

CP 114

ELASTIC

METHOD"

Fig. 16



SECTION 1-1

is not increased :-

1. Increase slab thickness t to 380 mm throughout
   then $v = \dfrac{16{,}6 \times 48 \times 1{,}15^2 \times 1000}{4 \times 880 \times (380-50)} = 0{,}91$ MPa

2. Increase the Col. size to 950 x 950
   then $v = \dfrac{(825-13)\,1000}{4 \times 1180 \times 190} = 0{,}91$ MPa

3. Provide Shear reinforcement eg. steel grillage
4. Increase the slab thickness only over the Cols by a slab drop 380 mm x 2000 x 2000
   This however changes the construction to "Flat Slab" and is not considered here.
5. A combination of 1, 2 or 3.

<u>Solution</u>: Increase the slab thickness to $t = 250$ mm and provide a steel shear-head over the Col. (Fig 17)

Revised Dead Load = 6,4 kN/m²
Live Load = 7,2 " "
Total 13,6 " "

$P_{ULT} = 13{,}6 \times 48 \times 1{,}15^2 = 865$ kN. →

Choose 2 no 150 x 75 [welded back to back

Total Shear Resistance $= 4 \times 150 \times 13 \times 93$
$= 725$ kN
$\approx 80\%\ P_{ULT}$

If length l of shear head = 2000
Shear stress at critical periphery :-

$v = \dfrac{(865-6)\,1000}{4 \times 2000 \times ,707 \times 200} = 0{,}76$ MPa →
$< 0{,}9$ MPa

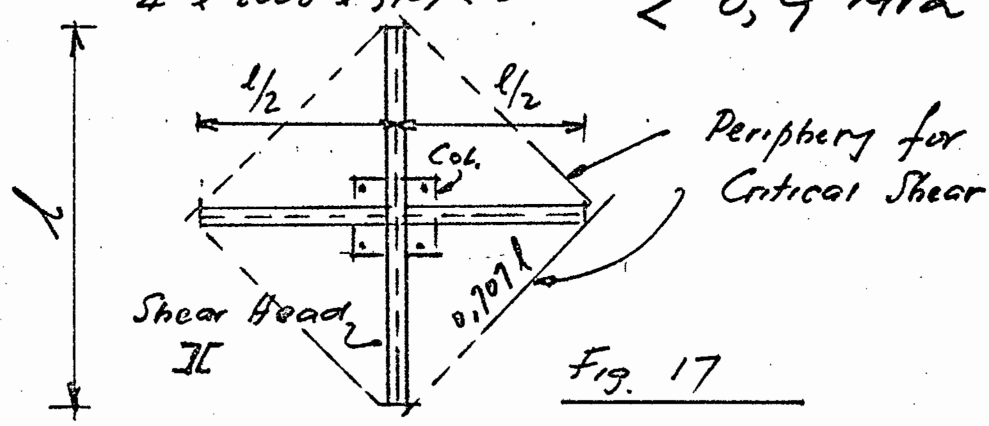


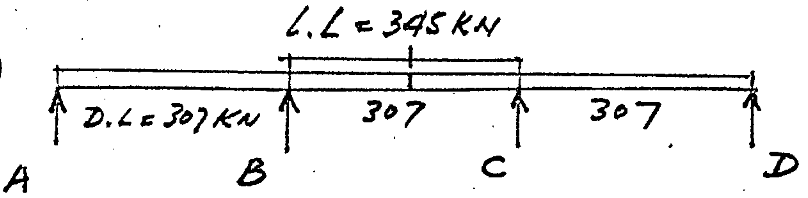
Fig. 17

Calculation of Max. Bending Moments (Fig 18)

(1) Frame ABCD Width = 6m, Span = 8m.

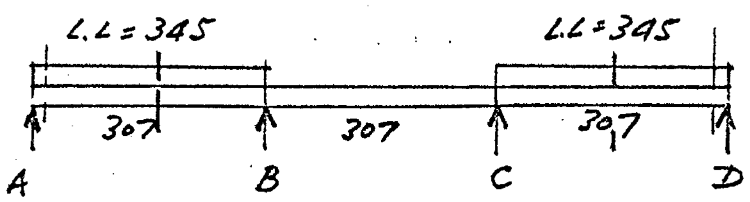
The following loading arrangements are considered:-

(a)



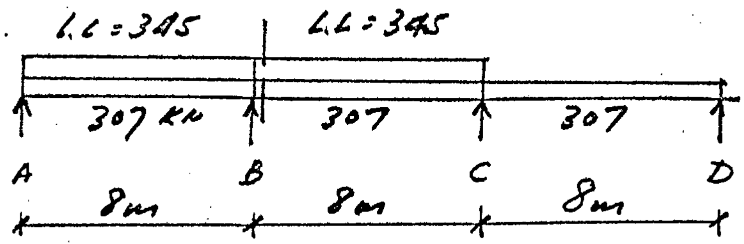
For Max B.M at B/C

(b)



For Max B.M at A/B, C/D and A, D

(c)



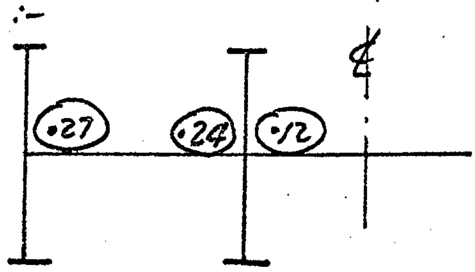
For Max B.M at B, C.

I_{slab} (Uncracked) $= \frac{1}{12} bt^3 = \frac{1}{12} \times 6000 \times 250^3 = 6000$ $\quad K_s = \frac{I}{L} = \frac{6000}{8000} = 0.75$

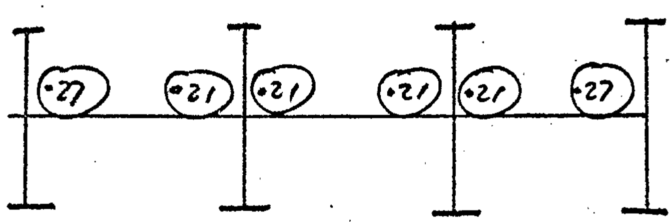
I_{col} (Uncracked) $= \frac{1}{12} bt^3 = \frac{1}{12} \times 500 \times 500^3 = 4000$ $\quad K_c = \frac{I}{L} = \frac{4000}{4000} = 1.00$

Distribution Factors for beams :-

Symmetrical Distribution
(a) (b)



Asymmetrical Distribution
(c)

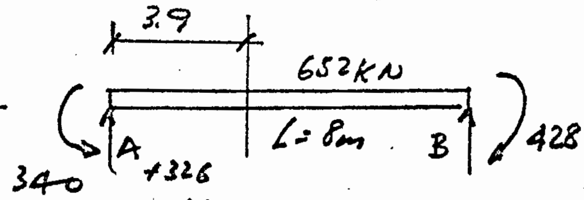


Fixed End Moments

Dead Load $M = 307 \times 8/12 = 205$ KN.m

Live Load $M = 345 \times 8/12 = 230$ KN.m a typical calculation will be carried out for (b)

	.27	.24	.12
D.L	-205	+205	-205
L.L	-230	+230	0
	+117	-55	-28
	-28	+59	0
	+8	-14	-7
	-7	+4	0
	+2	-1	0
	-340	+428	-240
Col. Moments	(+170)	(-94)	



+326
− 10
316 x = 3.9

$M_{A/B} = 3.9 \times 3.16 \times 1/2 - 340 = 278$ kN.m

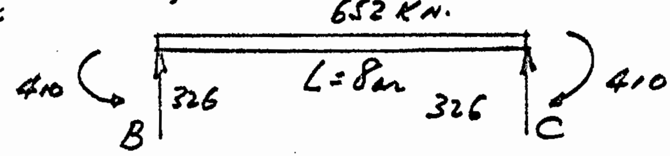
Critical $M_{AB} = 340 - 316 \times 0.375 = 222$ kN.m

and for (c)

	.27	.21	.21	.21	.21	.27
D.L	-205	+205	-205	+205	-205	+205
L.L	-230	+230	-230	+230	0	0
	+117	0	0	-47	-47	-55
	0	+59	-24	0	-28	-24
	0	-7	-7	+6	+6	+7
	-4	0	+3	-4	+4	+3
	+1	-1	-1	0	0	-1
	-321	+486	-464	+390	-270	+148
	A	B		C		D

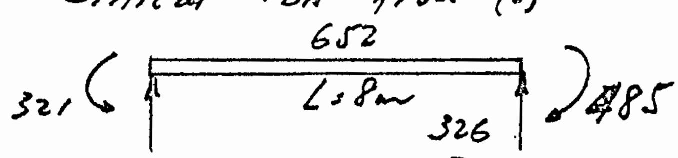
and for (a)

	.27	.24	.12
D.L	-205	+205	-205
L.L			-230
	+55	+55	+28
	+28	+28	0
	-8	-7	-3
	-3	-4	0
	+1	+1	0
	-132	+278	-410

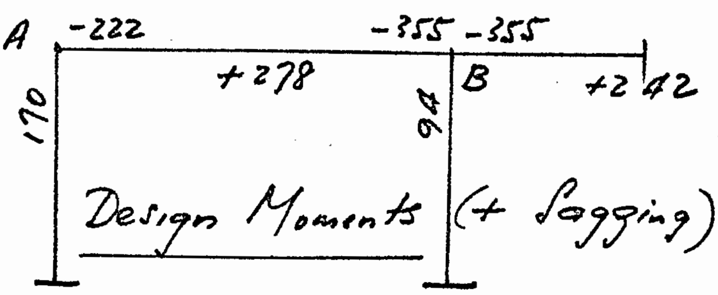


$M_{B/C} = 326 \times 4 \times 1/2 - 410 = 242$ kN.m

Critical M_{BA} from (c)



326
+ 20
346



$M_{BA} = 485 - 346 \times 0.375 = 355$ kN.m

Minimum $M_o = \cdot 10\, wL \left(1 - \frac{2c}{3L}\right)^2 = \cdot 10 \times 652 \times 8 \left(1 - \frac{2 \times \cdot 5}{24}\right)^2$

$= \cdot 10 \times 652 \times 8 \, (\cdot 96)^2 = \underline{480 \text{ KN.m}}$

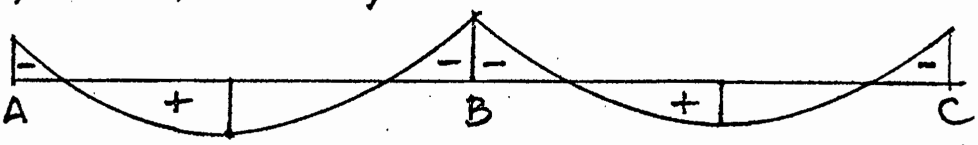
Numerical sum of moments for design :-

in AB $= \frac{222}{2} + 278 + \frac{355}{2} = 567$ KN.m $\longrightarrow$

in BC $= 355 + 242 \qquad = 597$ KN.m. $\longrightarrow$

both of which are $> M_o$ and satisfactory.

These Moments are now apportioned to the Column and Mid strips resp. as follows :-



			AB	A/B	BA	B/C
COL. STRIP	b = 3000 d = 210	%M	75	55	75	55
		M	−166	+153	−266	+133
		As	4450 mm²	3900	8100	3450 mm²
		*R	7Y20 + 7Y20	13Y20	6Y20 + 20Y20	11Y20
MID STRIP	b = 3000 d = 210	%M	25	45	25	45
		M	−56	+125	−89	+109
		As	1500	3200	2100	2750
		R	5Y20	10Y20	5Y20 + 5Y16	9Y20

* R = Reinforcement.

<u>Typical Calculation of Reinforcement Areas.</u>

BA - Col. Strip. $M = 266$ KN.m width $b = 3000$ $d = 250 - 40 = 210$ mm

$u_w = 25$ MPa $\quad c = 25 \times \frac{2}{3} \times \frac{1}{3} = 5{,}55$ MPa

$RM = 5{,}55 \times b \times \frac{d}{2} \times \frac{3}{4} d = 2{,}08\, bd^2$ N.mm.

$= 2{,}08 \times 3000 \times 210^2 \times 10^{-6} = 275$ KN.m. $\longrightarrow$

Lever arm $j = 0{,}75$ $\quad f_s = 30\,000$ psi $= 210$ MPa

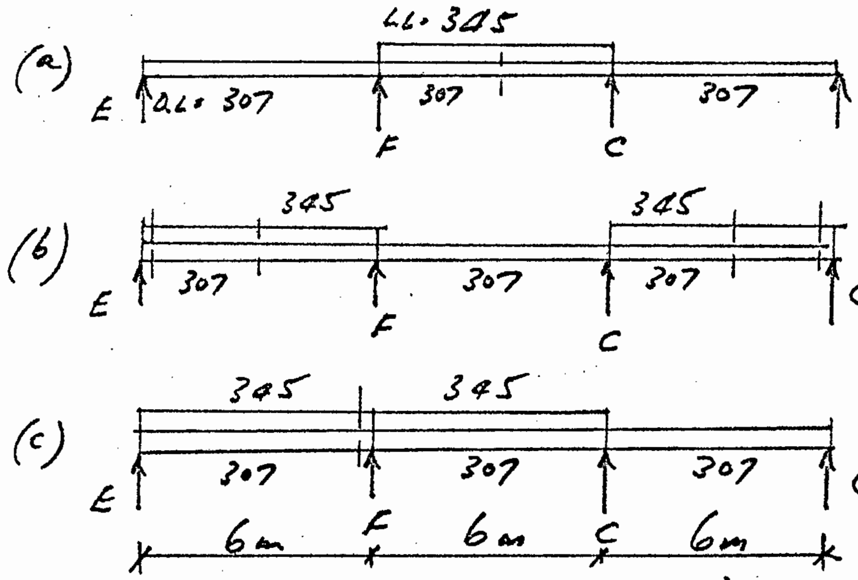
$Ast = \frac{M}{f_s \times j \times d} = \frac{266 \times 10^6}{0{,}75 \times 210 \times 210} = \underline{8100 \text{ mm}^2}$

A/B - Col. Strip $M = 153$ KN.m $\quad j = 0{,}88$ say

$Ast = 153 \times 10^6 / 0{,}88 \times 210 \times 210 = \underline{3900 \text{ mm}^2}$

(2) Frame E F C G Width = 8m, Span = 6m

The following loading arrangements are considered :-



(a) For Max B.M at F/C

(b) For Max B.M at E, E/F, C/G, G.

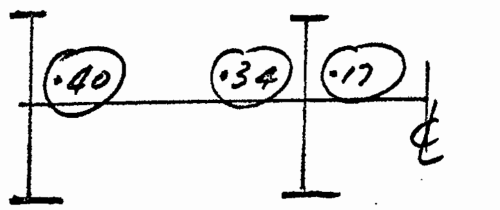
(c) For Max B.M at F, C.

$I_{slab} = 1/12\, bt^3 = 1/12 \times 8000 \times 250^3 = 8000$ $K_s = I/L = \frac{8000}{6000} = 1{,}33$

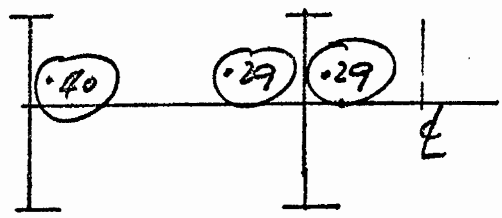
$I_{col} = 1/12\, bt^3 = 1/12 \times 500 \times 500^3 = 4000$ $K_c = I/L = \frac{4000}{4000} = 1{,}0$

Distribution Factors for beams :-

Symmetrical Distribution
(a) (b)



Asymmetrical Distribution
(c)

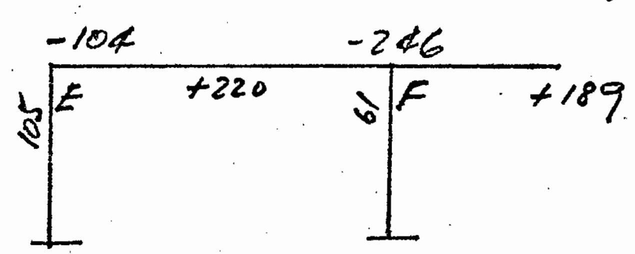


Fixed End Moments :-

Dead Load $M = 307 \times 6/12 = 154$ KN.m

Live Load $M = 345 \times 6/12 = 173$ KN.m.

The Moments are distributed as before resulting in the following design moments :-



$M_0 = .10\, wL \left(1 - \frac{2c}{3L}\right)^2$

$= .10 \times 652 \times 6 \left(1 - \frac{1{,}0}{18}\right)^2$

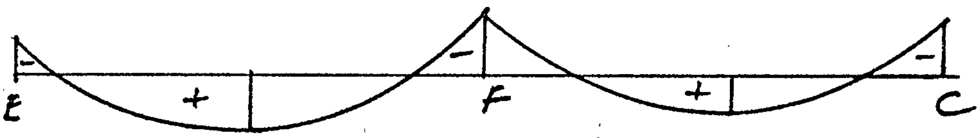
$= 350$ KN.m.

Numerical sum of moments for design :-

in EF $= \frac{104}{2} + 220 + \frac{246}{2} = 396$ KN. m ⟶

in FC $= 246 + 189 = 435$ KN. m ⟶

both of which are > M_o and therefore satisfactory. These panel Moments are now apportioned to the Col. and Mid strips with the resulting following steel Areas.



			EF	E/F	FE	F/C	
COL STRIP	b = 4000 d = 190	%M	75	55	75	55	
		M	– 76	+ 120	– 185	+ 104	KN. m
		As	2200	3400	5300	2900	mm^2
		R	9Y16 + 4Y16	18Y16	9Y16 + 18Y16	15Y16	
MID STRIP	b = 4000 d = 190	%M	25	45	25	45	
		M	– 28	+ 100	– 61	+ 85	KN. m
		As	700	2800	1700	2400	mm^2
		R	7Y16	14Y16	7Y16 + 7Y12	12Y16	

The Layout of Reinforcement for a full panel strip in both directions is shown in Fig 19. The bending shapes and lengths of all the bar types will be kept constant in all subsequent alternative designs having been found to satisfy the requirements with an economical volume of reinforcement. Subsequent requirements will be met by varying the numbers and diameters of the various types.

For the above design E1, the total weights of reinforcement including the Structural Steel Shear heads taken over all Columns is :–

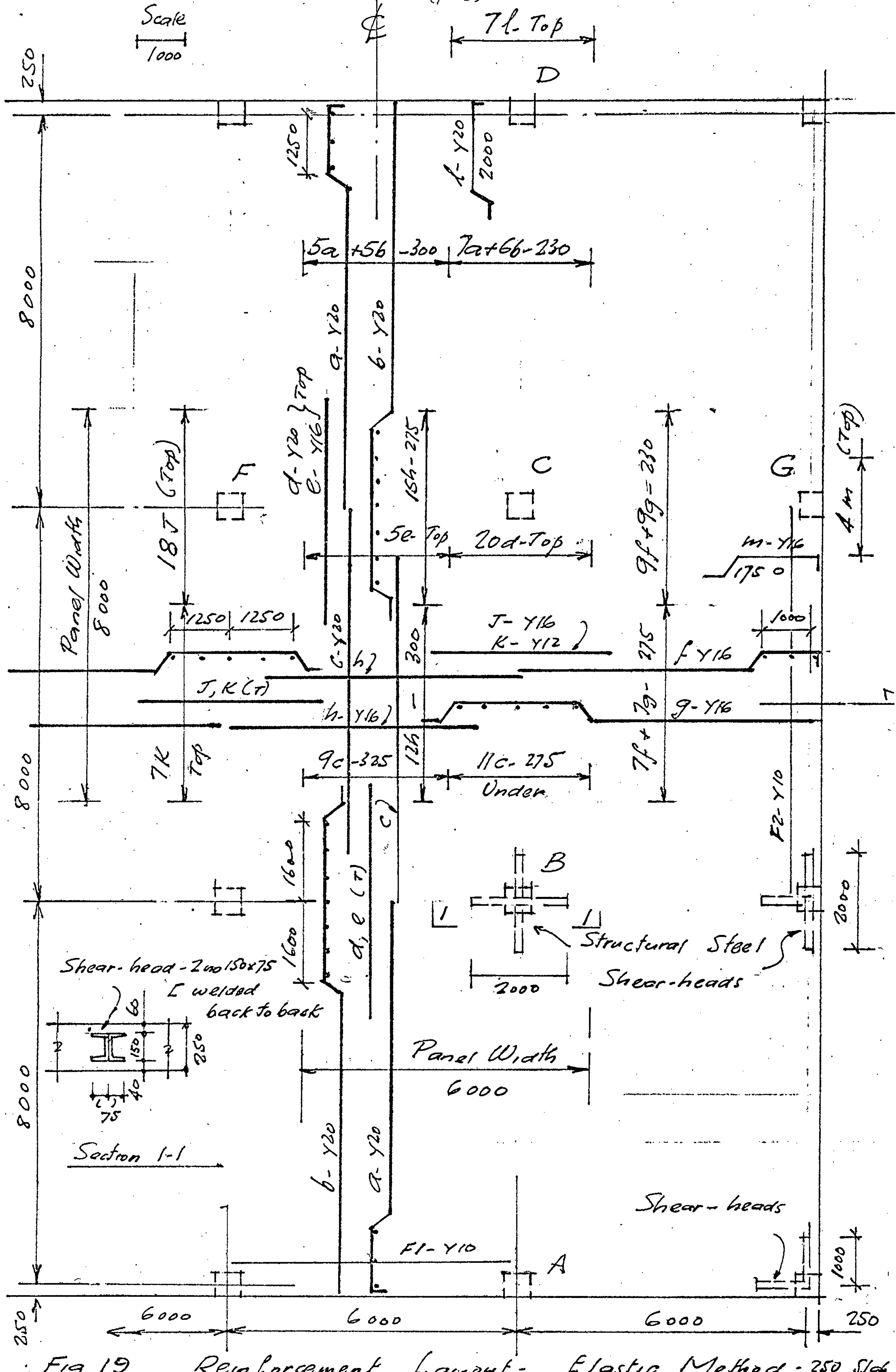


Fig 19 Reinforcement Layout - Elastic Method - 250 Slab.

Schedule of Steel Reinforcement for 1 Panel Strip both ways.

Mk	No	Dia	Length	Bending	Set
a	24	Y20	8,5	1500 270 6600	210
b	22	Y20	10,1	6200 270 3200 270	210
c	20	Y20	7,0	STRT	
d	4	Y20	4,8	STRT	
e	10	Y16	4,8	STRT	
f	32	Y16	6,5	1250 220 4850	170
g	32	Y16	7,7	4600 220 2500 220	170
h	27	Y16	5,3	STRT	
J	36	Y16	3,6	STRT	
K	14	Y12	3,6	STRT	
l	14	Y20	2,55	150 2000 270	210
m	8	Y16	2,25	150 1750 220	170
F1	20	Y10	6,0	STRT	
F2	16	Y10	8,0	STRT	

TOTAL WT = 3424 Kg.

WT. OF PANEL REINFORCEMENT = 3424 × 3 = 10272

" " " 150 × 150 ⌶ Shear heads = 1730

TOTAL MASS (Kg) = 12000

UNIT MASS = 12000/18 × 24 = 27,8 Kg/m²
(5,7 lb/ft²)

Optimum Slab thickness

In order to determine the most economical slab thickness, the mass of reinforcement for various other slab thicknesses all greater than the minimum of 250 was estimated, and the relative cost of each slab was assumed to be the cost of concrete and reinforcement per unit area. The cost of decking, columns etc. was assumed to be constant for all slabs.

The unit mass of steel reinforcement was assumed to be proportional to the total load (dead + live) and inversely proportional to the effective depth of the reinforcement in the long direction.

Assumed Unit Rates

Concrete in Slab (25 MPa) = R 16,00 per metre^3
Steel Reinforcement @ R250 per metric ton = R 0,25 per Kg.

Examples

i) $t = 300$ Steel mass $= 1730 + 10270 \times {}^{210}/_{260} \times {}^{14,8}/_{13,6} = 10,780$ Kg.

$= 25$ Kg/m^2 ⟶

ii) $t = 330$ Steel mass $= 1730 \times \frac{210}{290} + 10270 \times \frac{210}{290} \times \frac{15,4}{13,6} = 9.700$ Kg. $= 22,2$ Kg/m^2 ⟶

The length of the Shear-heads are reduced in inverse proportion to the effective depth of the slab

*iii) $t = 380$ Steel mass $= 0 + 10270 \times \frac{210}{340} \times \frac{16,6}{13,6} = 7750$ Kg.

$= 18$ Kg/m^2 ⟶

* Note. No shear reinforcement is required.

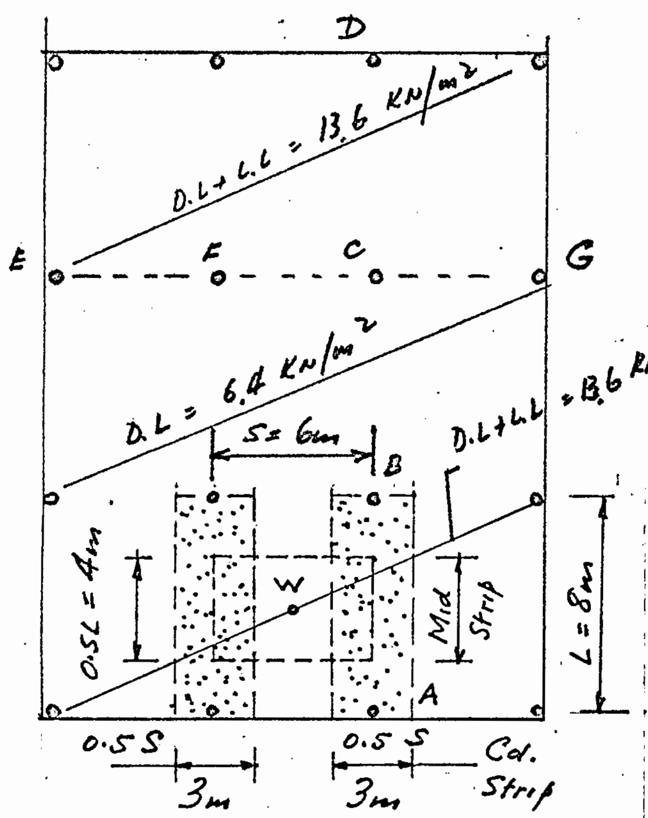
SLAB : t		250	300	330	350	*380	Rate
Cost R/m^2	Concrete	4,00	4,80	5,30	5,60	6,10	R16/m^3
	Steel	7,00	6,25	5,55	5,43	4,50	25¢/Kg
	TOTAL	11,00	11,05	10,85	11,03	10,60	

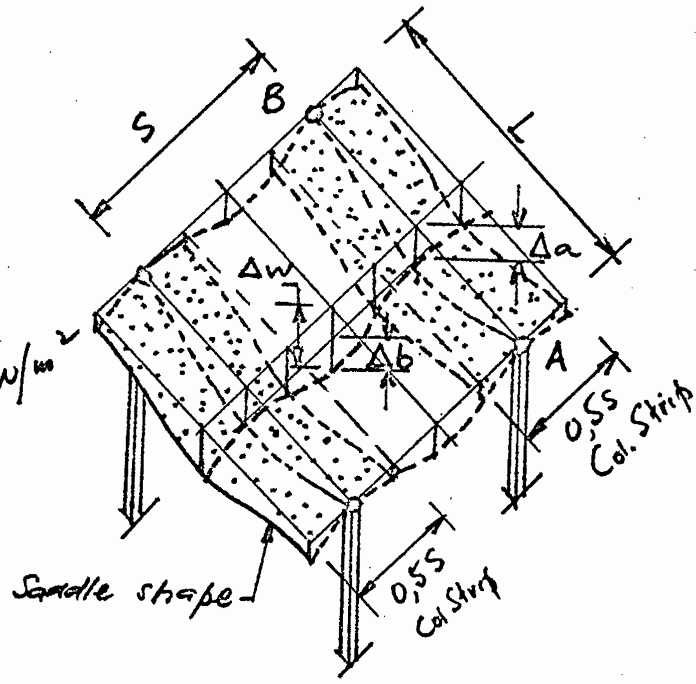
The most economical flat plate thickness is 380mm compared to a thickness of 250mm which satisfies the minimum depth and bending requirements of C.P. 114. Furthermore the deflections will obviously be negligible in comparison.

In practice however a uniform under surface is not essential for warehouse loading, and consequently the most economical slab will be a 250mm flat slab thickened to 380mm over the columns to form slab drops of at least $\frac{L}{4} = 2000 \times 2000$ at internal cols. No shear reinforcement is required, the concentration of reinforcement in the column strip over the columns will be reduced by the increased effective depth and furthermore deflections will be less than the 250 mm flat plate owing to the increased moment of inertia over the columns.

Estimation of Deflections

As no formula is recommended in CP 114 or ACI 318-63 for flat plates, the following approximate method is suggested by the writer based on recommendations in References 14, 15, 31, and Branson's Formula in Reference 39 for continuous R.C. beams in flexure. The deflection Δw is required at the centre point w of the external panel under the loading condition shewn :-





Deflected Shape of Panel

Fig. 21

w - centre point of Panel

Fig 20 - Loading arrangement, Deflection point W and Subdivision into Beam Elements of Column Strip Width and Mid Strip Width.

From Figs. 20, 21 $\underline{\Delta_w = \Delta_a + \Delta_b}$ where

Δ_a = deflection at the centre of the beam element in the long direction, of width assumed equal to the column strip, with respect to the support Columns.

Δ_b = deflection at the centre of the beam element in the short direction of width assumed equal to the mid-strip, with respect to its ends.

The choice of the widths of the beam elements is arbitrary but should nevertheless be made so that the distributions of B.M.'s across the critical sections are both nearly uniform and their total value can be closely estimated. In references, 14, 15, the full panel width is considered. In reference 31, a smaller width (0.4S) is chosen in the long direction for the calculation of EI, but the resulting B.M.'s are greater than those in the Column strips using CP.114. In the same reference 31, the deflection Δb was found from exact "finite difference" solutions, to be very nearly the deflection at the centre of a uniformly loaded rectangular slab of the same size clamped at all edges at the same level. For a square clamped slab, the centre deflection is

Ref. 31 $\Delta = \underline{0.00126\ qa^4/_D}$ and for a simply supported slab

Ref. 34 it is $\underline{0.00406\ qa^4/_D}$

The centre deflection of an interior panel of a square slab on point supports is however $\Delta = \underline{0.00581\ qa^4/_D}$ which is nearly 5 times the deflection of the slab having line supports.

The writer proposes to use the Moments calculated from the Elastic Analysis and to apportion them <u>not</u> in the arbitrary proportions laid down in the Codes, but in the proportions determined from the "exact" analysis of Brotchie and Russell in Ref. 4, Table 2. (See Elastic-Plastic Analysis). Another solution will be obtained later using the deflection of the Inner Column strip of width = $S/_4$ calculated in the same way from the B.M.'s determined by Russell in Ref. 20, Table 8. For simplicity, the rigidity of the beam element from the ℄ of the Col. to the face of the Col ($EI = \infty$) will be ignored, but this can be taken into account if desired. The end rotations can be determined from the calculated column moments and their stiffnesses. Fig. 22

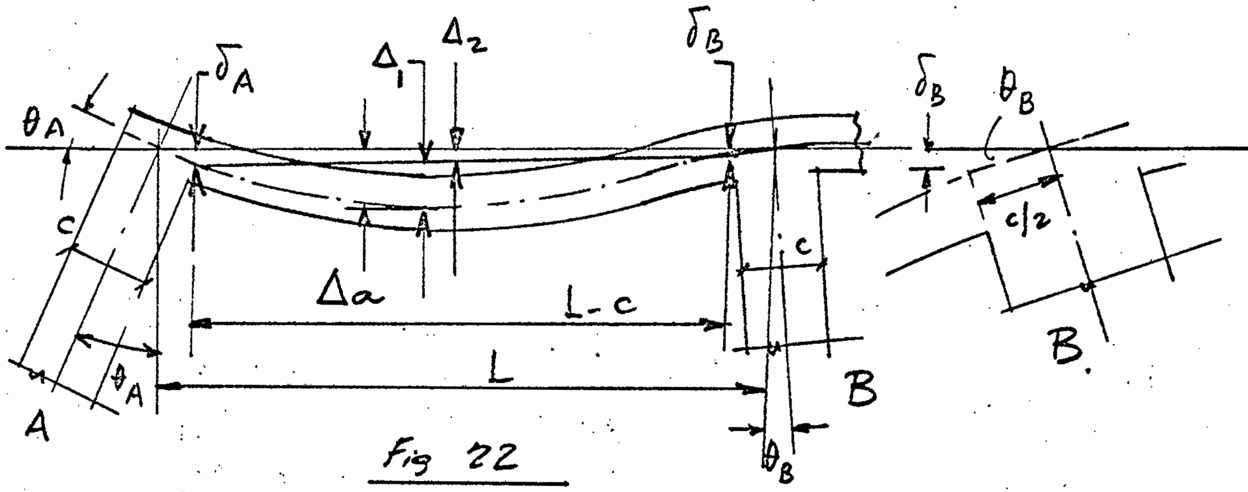


Fig 22

Then $\Delta a = \Delta_1 + \Delta_2$ where

Δ_1 = deflection in span $(L-c)$ calculated from end Moments M_A, M_B at face of Cols and $M_{B/C}$ the max. Moment near the centre of the span

$$\Delta_2 = \frac{1}{2}\delta_A + \frac{1}{2}\delta_B = \frac{c}{4}(\theta_A + \theta_B) \longrightarrow$$

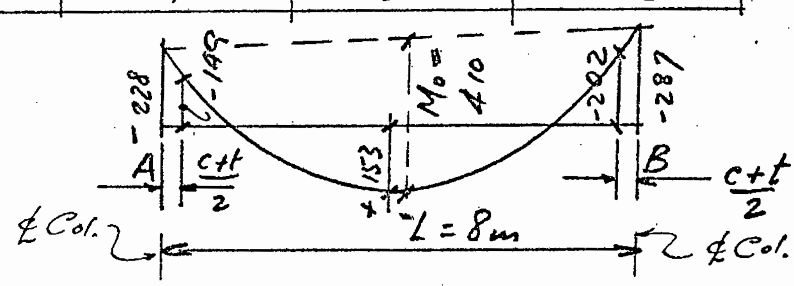
$$\theta_A = \frac{M_{Col\ A}}{4EI_{col}/L_{col}} \quad \text{etc.} \longrightarrow$$

Deflection Δa of beam element AB. Fig. 21

Ratio of Sides $= \frac{L}{S} = \frac{8}{6} = 1{,}33$ $\quad r_b = \frac{c}{\sqrt{\pi}} = 0{,}57c = 285$

$$\frac{r_b}{L} = \frac{285}{8000} = {,}035$$

MOMENTS	AB	B/C	BA	
Panel Moment	-340	+278	-428	
Col. Strip %	67%	55%	67%	
(Brotchie)	= -228	+153	-287	KN.m
Design M	-149	+153	-202	KN.m.



Conc. Strength $f_c' = 3000$ psi

Modulus of Rupture $f'_{cb} = 8\sqrt{f_c'} = 8 \times 55 = 440$ psi $= 3{,}0$ MPa →

Cracking Moment

$$M_{cr} = \frac{f'_{cb}\, I_g}{\frac{t}{2}} = \frac{3{,}0 \times 39 \times 10^8}{125} = 94 \text{ kN.m}$$

where $I_g = \frac{1}{12} \times 3000 \times 250^3 = 39 \times 10^8 \text{ mm}^4$

$\frac{t}{2} = \frac{250}{2} = 125$

$E_c = 57{,}600 \sqrt{f_c'} = 3 \times 10^6$ p.s.i $= 2{,}07 \times 10^4$ MPa

$m = \frac{E_s}{E_c} = 10$

Mid-Span $M_{max} = 153 > M_{cr}$ $A_s = 3900 \text{ mm}^2$

$A_s = 3900 \text{ mm}^2$ $p = \frac{3900}{3000 \times 210} = 0{,}62\,\%$

Ref 13 $E_c I_{cr} = .120 \times 10^6 bd^3$ for $E_c = 3 \times 10^6$ p.s.i

$\therefore I_{cr} = .04 \times bd^3 = .04 \times 3000 \times 210^3$

$= 110 \times 10^7 \text{ mm}^4$ →

Ref 14 $I_{cr} \approx 0.6 m \times A_s \times d^2 \approx 103 \times 10^7 \text{ mm}^4$

Ref 39 - Branson $I_{eff} = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr}$

$= \left(\frac{94}{153}\right)^3 \times 39 \times 10^8 + \left[1 - \left(\frac{94}{153}\right)^3\right] 11 \times 10^8$

$= 9{,}1 \times 10^8 + 8{,}5 \times 10^8$

$I_0 = 17{,}6 \times 10^8 \text{ mm}^4$ →

Support AB $A_s = 4450 \text{ mm}^2$ $I_{eff} = \left(\frac{94}{149}\right)^3 \times 39 \times 10^8 + \left[1 - \left(\frac{94}{149}\right)^3\right] 12{,}5 \times 10^8$

$= 10 \times 10^8 + 9{,}4 \times 10^8$

$I_1 = 19{,}4 \times 10^8 \text{ mm}^4$ →

Support BA $A_s = 8100 \text{ mm}^2$ $I_{eff} = \left(\frac{94}{202}\right)^3 \times 39 \times 10^8 + \left[1 - \left(\frac{94}{202}\right)^3\right] 23 \times 10^8$

$= 3{,}9 \times 10^8 + 20{,}7 \times 10^8$

$I_2 = 24{,}6 \times 10^8 \text{ mm}^4$ →

Span AB Average $I_{eff} = \frac{1}{2} I_0 + \frac{1}{4} I_1 + \frac{1}{4} I_2$

$=$ Say $20 \times 10^8 \text{ mm}^4$ →

Assume the distribution of B.M along the span AB is parabolic. m = average of support moments

$$= \frac{228 + 287}{2} = 257 \text{ KN.m}$$

M_o = simply supported B.M = 257 + 153 = 410 KN.m

$$\Delta a = \frac{5}{48}\frac{M_o L^2}{EI} - \frac{mL^2}{8EI} \longrightarrow$$

$E_c = 3 \times 10^6$ p.s.i = $2{,}07 \times 10^4 \text{ N/mm}^2$

$I_{eff} = 20 \times 10^8 \text{ mm}^4$ $\quad L = 8000$

$$\therefore \Delta a = \left(\frac{5}{48} \times 410 \times 10^6 - \frac{1}{8} \times 257 \times 10^6\right)\frac{L^2}{EI}$$

$$= \frac{10{,}6 \times 10^6 \times 8^2 \times 10^6}{2{,}07 \times 10^4 \times 20 \times 10^8}$$

$$= 16 \text{ mm} \longrightarrow$$

Deflection Δb of beam element in Short Direction

Fig. 21

MOMENTS	INT. SUPPT	CENTRE	INT. SUPPT	
Panel M	−338	+152	−338	$D+L = 13.6 \text{ KN/m}^2$
Mid Strip %	18%	29%	18%	
(Brotchie)	−61	+44	−61	KN.m
Design M	−39	+44	−39	

$$M_{cr} = \frac{f'_{cb} I_g}{t/2} \qquad I_g = \frac{1}{12} \times 4000 \times 250^3 = 52 \times 10^8 \text{ mm}^4$$

$= 125 \text{ KN.m} > M_{max}$.

$\therefore I_{eff} = I_g = 52 \times 10^8 \text{ mm}^4$ $\qquad M_o = 61 + 45 = 106 \text{ KN.m}$

$$\Delta b = \frac{5}{48}\frac{M_o L^2}{EI} - \frac{m l^2}{8EI}$$

$$= \left(\frac{5}{48} \times 106 \times 10^6 - \frac{1}{8} \times 61 \times 10^6\right)\frac{l^2}{EI}$$

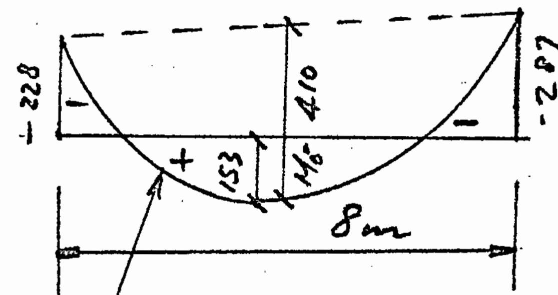
$$= \frac{3{,}5 \times 10^6 \times 6^2 \times 10^6}{2{,}07 \times 10^4 \times 52 \times 10^8}$$

$$= 1{,}2 \text{ mm}$$

$\therefore \Delta w = \Delta a + \Delta b = 16 + 1{,}2 = 17 \text{ mm} \longrightarrow$

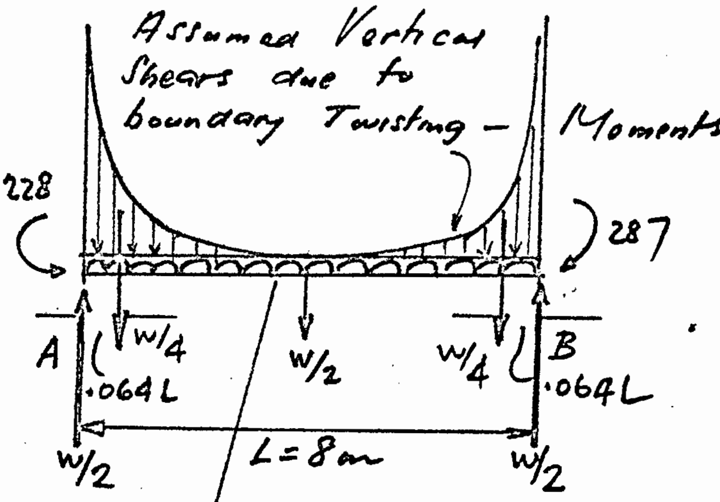
Short term Deflection/Span $= 17/8000 = 1/470 \longrightarrow$

The distribution of B.M along both Col. Strip and Mid-strip was assumed to be parabolic ie uniform vertical loading, in order to estimate Δa and Δb. However a more accurate assessment of the loading can be made, from which the deflection can be determined as follows:-



Ass. Parabolic Distribut^n

Previously Assumed B.M. Diagram



Uniformly distributed super-load $= w/2$

Loading Diag. on Col. Strip AB.

$M_0 = \dfrac{228+287}{2} + 153 = \underline{410 \text{ KN.m}}$

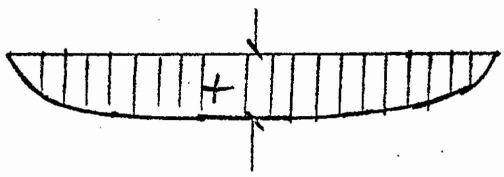
$M_0 = \dfrac{w}{2} \times \dfrac{L}{8} + \dfrac{w}{4} \times 0{,}064L$

$= 0{,}0785\, wL$

$= 0{,}0785 \times 652 \times 8$

$= \underline{410 \text{ KN.m}}$

$M = .016wL = 84 \text{ KN.m}$

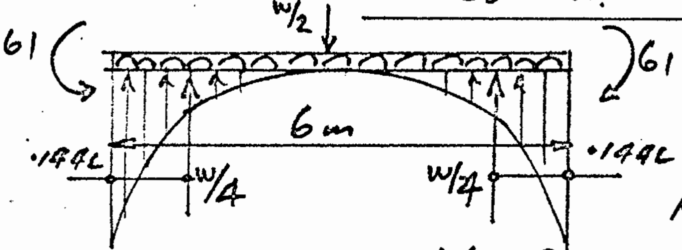


B.M. Diag due to Twisting Moments. $\approx$ Constant.

Revised $\Delta a = \left(\dfrac{5}{48} \dfrac{\times 652 \times 8}{16} + \dfrac{1}{8} \times 84 - \dfrac{1}{8} \times 237\right) \dfrac{L^2 \times 10^6}{EI}$

$= 12{,}5 \times 10^6 \dfrac{L^2}{EI}$

$= \underline{20 \text{ mm.}}$



Loading Diag. on Mid Strip

$M_0 = \dfrac{w}{4}\left(\dfrac{L}{4} - .144L\right) = .027\, wL$

$= .027 \times 652 \times 6 = 105 \text{ KN.m}$ ✓

Revised $\Delta b \doteqdot 0$.

$\therefore \underline{\Delta w = 20 + 0 = 20 \text{ mm}}$

Long- Term Deflection

Short Term D.L. deflection = 10 mm approx

Creep and Shrinkage

Long term deflection = 2x10 = 20 (A.C.I Recomm)

Short Term L.L deflection = 11

Total Long Term Deflection = 41 mm = $\frac{Span}{200}$.

EE2. EMPIRICAL METHOD

Slab thickness for fy = 60.000 psi $t = \frac{L}{30} = \frac{8000}{30} = 280$ mm

Dead Load - Slab = 2400 × 280 Kg/m²
= 6,6 KN/m²

Finish = ,5

Live Load = 7,2

Total = 14,3 KN/m²

P_{INT} = 14,3 × 6×8 × $1,15^2$ = 910 KN.

P_{ULT} : 1.6 × 910 = 1450 KN.

$V_{ult} = \dfrac{1450}{4\left[500 + (280-50)\right]230} = 2,16$ MPa

Max V_{ult} = $4\sqrt{f'_c}$ psi = $4\sqrt{3000}$ = 220 psi = 1,52 MPa

As previous method, provide Shear Reinforcement

ALTERNATELY

Increase t to 380 P_{ULT} = 1,6×1050 = 1680

$V_{ULT} = \dfrac{1680}{4\left[500 + (380-50)\right]330} = 1,53$ MPa ✓

BENDING MOMENTS IN LONG DIRECTION

Numerical sum $M_o = 0.10\,WLF\left(1-\frac{2c}{3L}\right)^2$

W = 14,3 × 6 × 8 = 686 KN

c = 500 L = 8000

$F = 1,15 - \frac{c}{L} = 1,09$

$M_o = 0.10 \times 686 \times 8 (1 - .04)^2$
= 550 KN.m →

			AB	A/B	BA	B/C
COL STRIP	b: 3000 d: 240	%Mo	40	28	50	22
		M	-220	154	-275	121
		As	5200	3500	6800	2700
		R	6Y20 + 10Y20	11 Y20	5Y20 17Y20	9Y20
MID STRIP	b: 3000 d: 240	%Mo	10	20	18	16
		M	-55	110	-99	88
		As	1200	2400	2200	2000
		R	4Y20	8Y20	4Y20 5Y16	7Y20

Typical Calculation of Reinf. Areas

BA- Col. Strip $M = 275$ KN. m

R.M (Conservative CP114) $= 2.08\, bd^2 = 360$ KN. m.

$j = 0.80$ $f_s = 210$ MPa

$$A_s = \frac{M}{f_s \times j \times d} = \frac{275 \times 10^6}{0.80 \times 210 \times 240} = 6\,800 \text{ mm}^2$$

BENDING MOMENTS IN 6m DIRECTION.

Numerical Sum M_o $= 0.10 \times 686 \times 6 \left(1.15 - \frac{500}{6000}\right) \times \left(1 - \frac{1}{18}\right)^2$

$= 400$ KN. m

			EF	E/F	FE	F/C
COL. STRIP	b: 4000 d: 220 J = 0.9	%Mo	40	28	50	22
		M	-160	112	-200	88
		As	3800	2700	4800	2100
		R	7Y16 + 8Y16	14Y16	7Y16 + 17Y16	11 Y16
MID STRIP	b: 4000 d: 220 0.9	%Mo	10	20	18	16
		M	40	80	72	64
		As	1000	1900	1700	1500
		R	5Y16	10Y16	5Y16 + 4Y16	8Y16

The reinforcement was then scheduled and weighted as previously. (E1)

Wt. of Panel Reinforcement : 3140 × 3 = 9 420
" " " 150 × 150 I = 1 730
Shear heads

Total Mass (Kg) = 11, 150

Unit Mass : $\frac{11\,150}{18 \times 24}$ = 26,0 Kg/m^2

Cost per m^2 : Reinforcement 26,0 × 0,25 = R 6-50
Concrete 280 × 16,00 = R 4-50
Total : R11,00

Comparison of Overall Cost : CP114 and ACI

Both methods E1 E2 result in designs which have the same overall cost for a large range of slab thicknesses

Comparison of Column Bending Moments

Column	Elastic E.1 CP114 t = 250	Empirical E.2 ACI t = 280
External Col	172	$\frac{wL}{30} = 686 \times 8/30 = 183$
Internal Col	94	$\frac{686 \times 8 - 343 \times 8}{40} = 69$

Conclusions

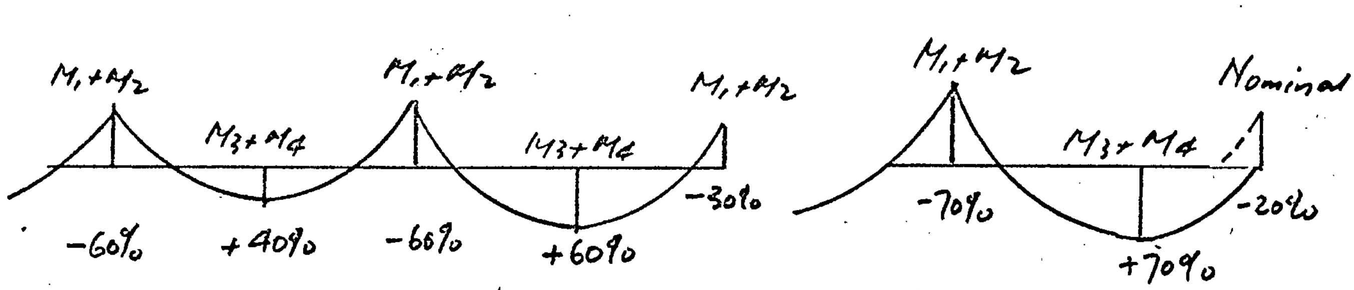
1) As both the Elastic Method and the Empirical Method yield equivalent designs, the latter method is recommended where applicable, as it is far quicker.
2) The optimum thickness of slab both for overall economy and acceptable deflections, is far greater than the minimum thickness required for bending alone.
3) Design of the slab thickness to satisfy the criterion of punching shear stress without shear reinforcement in this example, will produce the optimum slab thickness.

EE.3 YIELD LINE METHOD

As explained in an earlier Section (B) each Panel has an Ultimate Load for each direction, and therefore they should be equalised for economy.
The Ultimate Load in any direction is proportional to the numerical sum of the Positive and Negative yield moments across the entire width of Panel at the critical positive and negative moment sections.

i.e. $P_{ULT} \propto M_0 \propto M_1 + M_2 + M_3 + M_4$

where M_1, M_2 are the negative yield moments in the Col. strip and mid-strip resp. and
M_3, M_4 are the positive yield moments in the Col. strip and mid strip resp.
Note that M_1, M_2 etc. can be chosen arbitrarily. However the ratio $M_1 : M_2 : M_3 : M_4$ will control the deflection behaviour of the Panel after wide-spread cracking has occurred as well as the Volume of steel reinforcement for a given depth of slab. Therefore the following distribution of M_0 is recommended :-



COL.	-60%	+20%	-60%	+30%	-30%	-70%	+35%
MID	0	+20%	0	+30%	0	0	+35%

Stiff Cols $\dfrac{\Sigma K_{COL}}{K_{panel}} > 0.5$ → | ← Slender Cols.

Note : 1) No reinforcement for Neg. M in the mid-strips
2) Uniform distribution of reinft in Pos. M regions.

NOTE : 3) Provided the Factor of Safety for the Ultimate Load ie $\frac{\text{Ultimate Load}}{\text{Total Service Load}}$ is the same as the F. S. for the stress in the reinforcement, the Load Factor Method may be used in conjunction with the Total Service Load to determine the Areas of steel reinforcement.

PREVIOUS EXAMPLE

BENDING MOMENTS IN LONG DIRECTION

Slab thickness from Shear criterion = 380 mm →

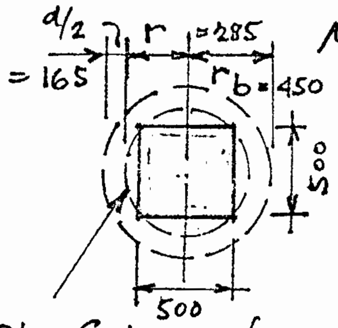
Dead Load – Slab = 2400×380 kg/m²
= 8,9 KN/m²

Finish = ,5

Live Load = 7,2

Total = 16,6 KN/m²

Panel Load = $16,6 \times 6 \times 8$ = 800 KN. →



Olar Column of equal area.

$M_o = \frac{PL}{8} - \frac{P r_b}{\pi}$ $L = 8m$ $^* r_b = 0,285 + \frac{0,330}{2}$

$= \frac{800 \times 8}{8} - \frac{800 \times 0,450}{\pi} = 800 - 115$

$= 685$ KN.m →

$^* r_b = r + d/2$

			AB	A/B	BA	B/C
COL STRIP	b=3000 d=340	%Mo	30	30	60	20
		M	-205	+205	-410	+137
		As	3200	3200	6400	2150
		R	11Y20	11Y20	20Y20	11-Y16
MID STRIP	b=3000 d=340	%Mo	0	30	0	20
		M	0	+205	0	+137
		As	0	3200	0	2150
		R	0	11Y20	0	11-Y16

Typical Calculation of Reinforcement Areas.

BA - Col. Strip $M = 410$ KN.m $d = 340$ $j = 0.9$ say

$f_s = 210$ MPa

$$A_s = \frac{M}{f_s \times j \times d} = \frac{410}{210 \times 0{,}9 \times 340} = \underline{6400 \text{ mm}^2}$$

BENDING MOMENTS IN SHORT DIRECTION

Panel Load $P = 800$ KN.

$$M_o = \frac{PL}{8} - \frac{P r_b}{\pi} \qquad L = 6\text{m} \quad r_b = 0{,}450$$

$$= \frac{800 \times 6}{8} - \frac{800 \times 0{,}450}{\pi} = 600 - 115$$

$$= \underline{485 \text{ KN.m}}$$

			EF	E/F	FE	F/C
Col. STRIP	b=4000 d=320	%Mo	30	30	60	20
		M	-146	146	-290	100
		As	2400	2400	4800	1600
		R	12Y16	12Y16	24Y16	12Y16
MID STRIP	b=4000 d=320	%Mo	0	30	0	20
		M	0	146	0	100
		As	0	2400	0	1600
		R	0	12Y16	0	12Y16

The layout of Reinfmt. is similar to Fig. 19* and Wt. of Steel reinforcement = 2600 x 3 = 7800 Kg.

This is exactly the same mass as estimated for $t = 380$ mm using the Elastic Method.

* excluding bars a, b, f, g, now straight.

Conclusions :-

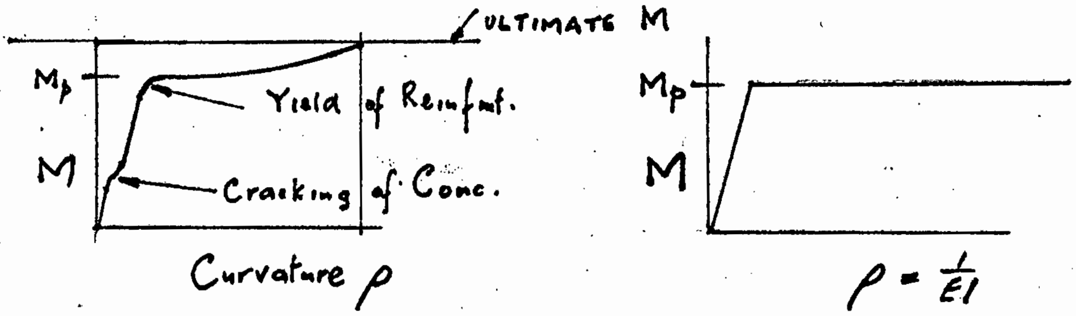
1.) Volume of Steel reinforcement determined by using Yield Line Theory is practically identical to that determined from either the Elastic or the Empirical Method for the same slab thickness.

2) Owing to the simplicity of its application, Yield Line Method will be prefered by those designers who have sufficient understanding of flat plate behaviour to be able to make subjective steel adjustments.

F. ELASTIC-PLASTIC ANALYSIS AND DIRECT DESIGN

F 1.1. ELASTIC-PLASTIC ANALYSIS

In the range of reinforcement ratios practicable for slabs, the moment-curvature relationship is shown in Figure (23) below and is conventionally approximated for analysis by the idealised curve shown :-



(a) Actual (b) Idealised

Figure 23 - Moment Curvature Relationship

If Mp is the yield moment of the slab per unit width i.e. the internal moment at the start of yield of the reinforcement, it can be predicted with reasonable accuracy by

$$Mp = pf_y d^2 \left(1-0.59 \frac{pf_y}{fc^1} \right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

As the idealised curve is similar to that for a homogeneous plate, the solution obtained can be extended to reinforced concrete slabs using the above equation.

Two-Dimensional Solution for an Internal Panel

Consider a uniformly loaded reinforced concrete plate on circular columns at regular intervals and unbounded in plan. The solution for this plate can be considered to apply to an internal panel. A substitute problem is first introduced. An auxiliary elastic supporting medium is placed in contact with the plate and all the column reactions are applied as external forces. When a single unit column reaction is applied, the slab deflects into the medium which exerts a reaction on the slab for which moments, shears and deflections can be calculated by the solution of the modified biharmonic equation for a plate on an elastic foundation i.e.

$$D \nabla^4 \omega + \epsilon \omega = q \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

with appropriate boundary conditions, where the notation is conventional and ϵ is a small term representing the reaction modulus of the medium.

In the Elastic range and beyond but provided yielding is confined to a line around the column, the surrounding

plate /

plate remains elastic and the solution for the deflection w is given by

$$w = C_1 Z_1 + C_2 Z_2$$

in which C1 and C2 are constants and Z_1 and Z_2 are tabulated functions of the radial ordinate which has its origin at the column centre. C_1, C_2 are evaluated from the boundary conditions at the column edge. C1 depends on the shear at the column edge and is proportional to the panel load throughout. C_2 is determined from the degree of rotational restraint at the column edge and therefore varies after radial yield. At any point the radial moment Mr and the tangential moment Mt can also be determined as a function of the radial ordinate alone. Provided yielding is confined to a line around the column edge, the total moments Mx,My at any point x, y in rectangular co-ordinates are then given by the superposition of sets of axisymmetrical components resulting in the following :-

(R20)

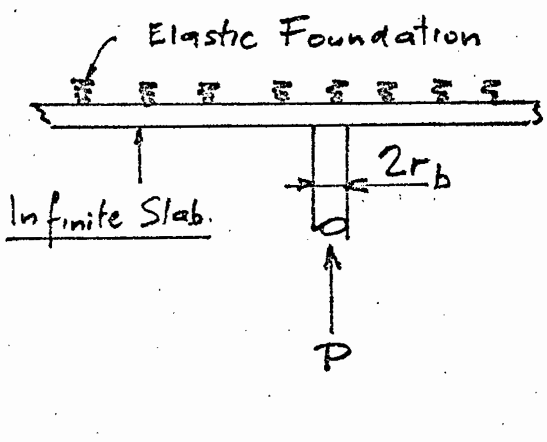
$$Mx = \sum_{r=b}^{\infty} M_r \cos^2\theta + M_t \sin^2\theta \quad \text{.........(48)}$$

$$My = \sum_{r=b}^{\infty} M_r \sin^2\theta + M_r \cos^2\theta \quad \text{.........(49)}$$

where θ = angular co-ordinate of the column from the point referred to the x - axis, b = column radius.

The uniform loading on the unbounded plate produces a rigid body translation and therefore has no additional effect. Figure (24) shows the moments in the component problem - a single axially loaded column on an elastically supported slab at various stages of yield. Poissons ratio = o.

(R4)



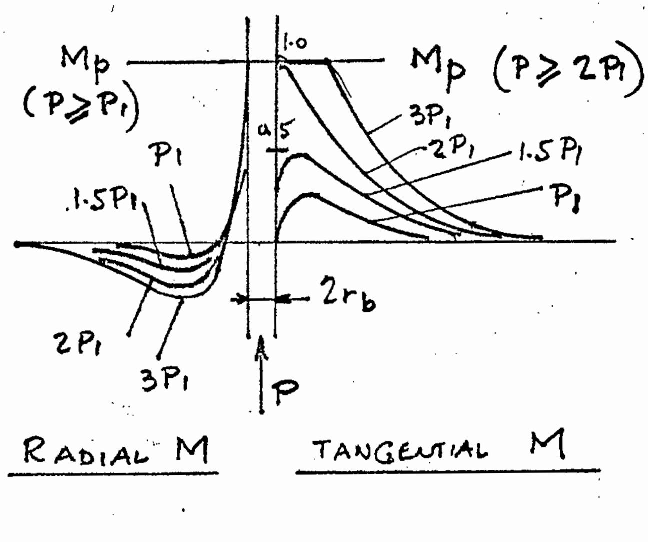


Figure 24 (a) Figure 24(b)

P1 = maximum elastic load = load causing radial yield of isotropic reinforcement with yield moment Mp.

The /

In the case of a square panel

$$P_1 = \frac{-2\pi M_p}{\text{Log}\, r_b/L + 1.31} \quad \ldots\ldots\ldots\ldots\ldots\ldots(50)$$

$$P_2 = \frac{-4\pi M_p}{\text{Log}\, r_b/L + 1.31} \quad \ldots\ldots\ldots\ldots\ldots\ldots(51)$$

$$P_3 = \frac{16 M_p}{1 - 8r_b/\pi L} \quad \ldots\ldots\ldots\ldots\ldots\ldots(52)$$

$$P_4 = \frac{16 M_p}{(1 - 2r_b/L)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(53)$$

P_3 and P_4 are determined from statics and Yield-Line theory. For a rectangular panel Mp is non-uniform over the columns and is given in each direction at the load P_2, by

$$M_{px} = \frac{P}{4\pi}\left[\text{Log}\,\frac{r_b}{L_x} + 1.310\right] \quad \ldots\ldots\ldots\ldots(54)$$

$$M_{py} = \frac{P}{4\pi}\left[\text{Log}\,\frac{r_b}{L_x} + 2.310 - \frac{L_y}{L_x}\right] \quad \ldots\ldots\ldots\ldots(55)$$

Ly = Long direction, Lx = short direction

A summary of the slab behaviour is presented below under load increasing to the Ultimate :-

Panel Load	Column Edge Mr	Column Edge Mt	Behaviour of Panel
$0 \longrightarrow P_1$	< Mp	0	Elastic
P_1	Mp	0	Elastic-Radial yield at Col.
$P_2 = 2P_1$	Mp	Mp	Elastic-Radial and Tangential yield at Column
$P_2 \longrightarrow P_3$	Mp	Mp	Elastic-Plastic: Tangential yield spreads outward
$P_3 \doteq 5P_1$ (Ultimate)	Mp	Mp	Elastic-Plastic: First yield pattern complete
$P_4 \doteq 5.35P_1$ (Collapse)	Mp	Mp	Elastic-Plastic : Modified yield pattern complete

The Moments in a uniformly loaded square internal panel at various stages of yield for constant Mp, $r_b = 0.05L$ is shown in Figure A.F14 of the appendix.

<u>Panel Moments at Load P_2</u>

At P_2, Mr = Mt = Mp, hence constants C_1 C_2 are known and the moments and deflection are readily determined throughout.

The /

The behaviour in the region of any internal column of a flat plate under increasing load closely follows the behaviour of the slab in the component problem. If the slab is completely restrained by the column, maximum moment occurs at the column edge in the radial direction i.e. Mr is the maximum moment. Hence yielding commences when Mr becomes = Mp, where Mp is the yield moment in all directions, at a load Pl say. Up to this stage Mt = o. A yield line forms about the edge and axisymmetric plastic rotation about this line allows radial slope and tangential moment to increase until Mt = Mr = Mp at the load $P_2 = 2P_1$ which is the maximum load for which yielding is confined to the column edge. At greater loads than P_2, tangential yield spreads concentrically outward to cover an increasing area of plate with radial cracks. When the load P_3 is reached yield has occured along all sections of principal moment i.e. along the column axes and the centre lines of the panels. Since at load P_3 the yield pattern is completed the ultimate load of the panel has been reached. The collapse load Pu is reached soon after with a modified yield pattern. Figure 25 shows the various stages of yield.

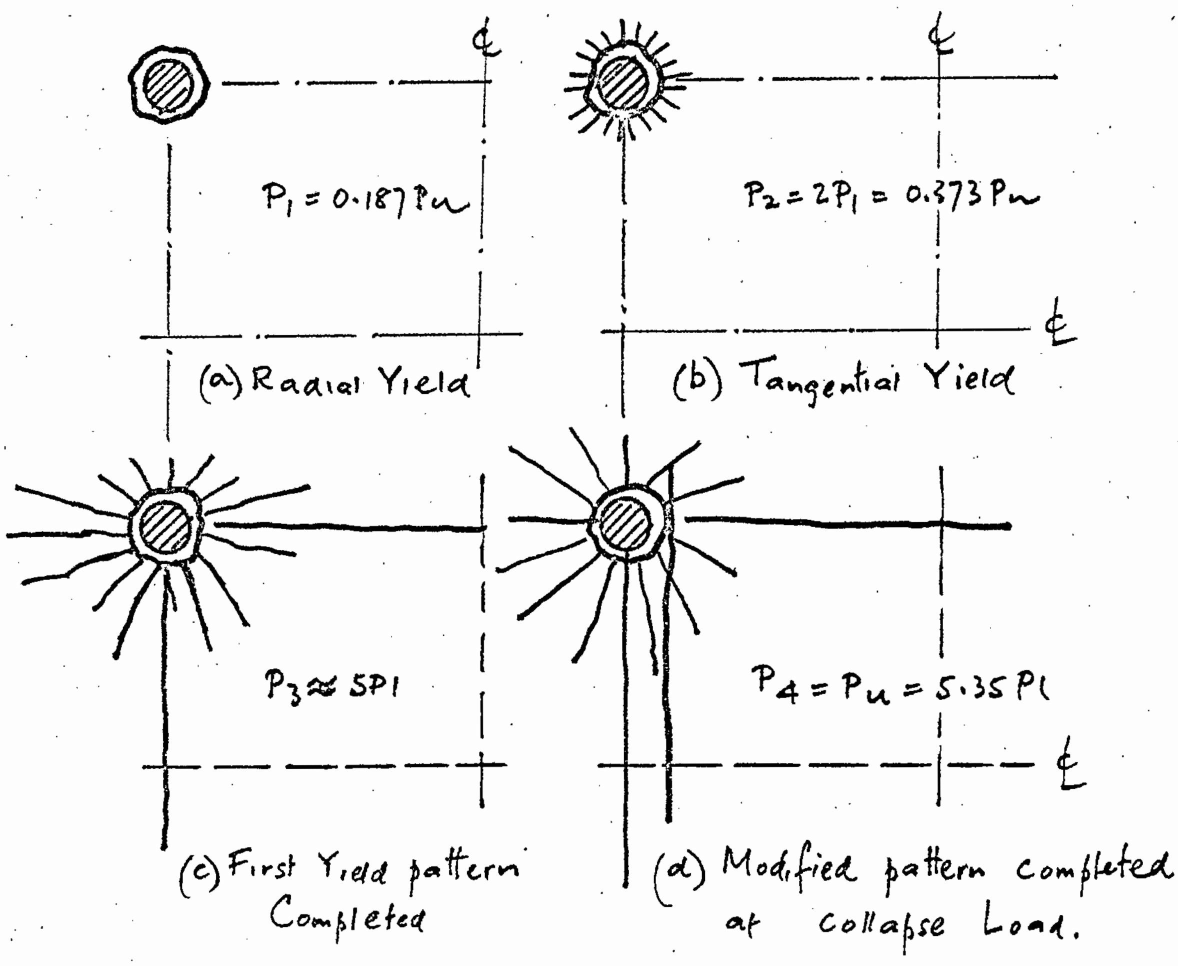


Figure 25 - VARIOUS STAGES OF YIELD

POSITIVE YIELD — — — —

NEGATIVE YIELD ————

In the /

The distribution of moments will be considered as a criterion for optimum behaviour for the following reasons :-

1. For any desired ultimate load P_3, the sum of the yield moments at the critical positive and negative moment sections is given by

   ....(56)

   $$Mo = m_1 + m_2 = \frac{P_3 L}{8} \left(1 - \frac{8r_b}{\pi L}\right)$$ and is therefore constant.

   As the area of reinforcement is approximately proportional to the yield moment, the total area of reinforcement at the two critical sections is nearly constant and can be calculated from P_3.

2. If Mo is distributed amongst the Column and mid strips of the critical sections in proportion to the <u>integrated</u> actual moments in these regions at P_2, which moments as stated can be exactly determined, it must follow that when the panel load equals P_2, yield moments will be reached and hence fracture lines will form simultaneously at all critical moment sections. The lengths of fracture will be limited however as it is not practical to vary the reinforcement to fit the moment curve exactly. Before P_2 is reached, the behaviour of the slab is elastić with minimum deformation.

3. For loads greater than P_2, yielding spreads rapidly causing plastic deformation and P_3 is soon reached.

4. Hence the Elastic Loading range approaches a maximum and the plastic range becomes a minimum for the given Ultimate Load P_3. This is illustrated in the following example for an interior panel of a square plate.

   $\frac{rb}{L} = 0.5$. From Table A.T1 in the Appendix, proportions of Mo are as follows :-

	Negative Moment		Positive Moment	
	Col. Strip	Mid Strip	Col. Strip	Mid Strip
Width	$L/_2$	$L/_2$	$L/_2$	$L/_2$
% Mo	45	17	23	15
Recommended Mp	$\boxed{1.0m}$	0.38m	0.5m	0.33m
Total M	0.5mL	0.19mL	0.25mL	0.16mL

$$Mo = \sum mL = mL\ (0.5 + 0.19 + 0.25 + 0.16) = mL \times 1.10$$

$$\text{But } Mo = \frac{PL}{8}\left(1 - \frac{8r_b}{\pi L}\right)$$

$$\text{Therefore } P_3 = 10\ m \longrightarrow$$

$$\text{From } (51)\ P_2 = \frac{-4\pi m}{\text{Log}_e \frac{r_b}{L} + 1.31} \doteqdot 7\ m \longrightarrow$$

$$\text{Therefore } \frac{P_2}{P_3} = 70\% \longrightarrow$$

By /

By comparison Mp was made uniform throughout so that

$$Mo = mL \times 1.10 \text{ for ultimate load } P_3$$

$$\text{Therefore } Mp = 0.55 \text{ m}$$

$$\text{From (51) } P_2 = 3.85 \text{ m and } \frac{P_2}{P_3} = 38.4\,\% \longrightarrow$$

At the other extreme the ratio P_2/P_3 can be increased still further simply by subdividing the column strip into inner and outer strips of width $= L/4$ and providing 50% of the total Negative yield moment in the inner column strip as follows:

	Negative Moment			Positive Moment	
	Col. Inner	Col. Outer	Mid Strip	Col. Strip	Mid Strip
Width	$L/4$	$L/4$	$L/2$	$L/2$	$L/2$
% Mo	50% x 62 =31	14	17	23	15
Recommended Mp	$\boxed{1.38m}$	0.62m	0.38m	0.5m	0.33m
M	0.345mL	0.155mL	0.19mL	0.25mL	0.16mL

$$Mo = \sum mL = mL \times 1.10 \quad \text{as before}$$

$$\text{From(51)} P_2 = 9.6 \text{ m and } P_2/P_3 = 96\% \longrightarrow$$

F.12. SUMMARY

1. If a square slab is reinforced isotropically to provide a given ultimate panel load equal to P_3 say, then plastic deformation in the panel will occur at loads $> P_2$ which is approximately 40% of P_3. For a load-factor of 2 or less this represents plastic deformation within the range of working load.

2. If the slab is reinforced so that the total yield moment in each strip is proportional to the integrated moment in the same strip and the moment in the inner column strip is proportional to 50% of the total negative moment which occur at the load P_2 corresponding to yield in all directions at the column edge, then P_2 approaches 96% of P_3 so that the behaviour will be elastic for almost the complete loading range. In this way, the behaviour of the slab will be optimized at working loads and large overloads. By minimizing immediate deflection, long term deflection due to creep and shrinkage will also be minimized.

3. If At represents the constant sum of reinforcement required, the distribution of steel and weights per panel will be as follows:-

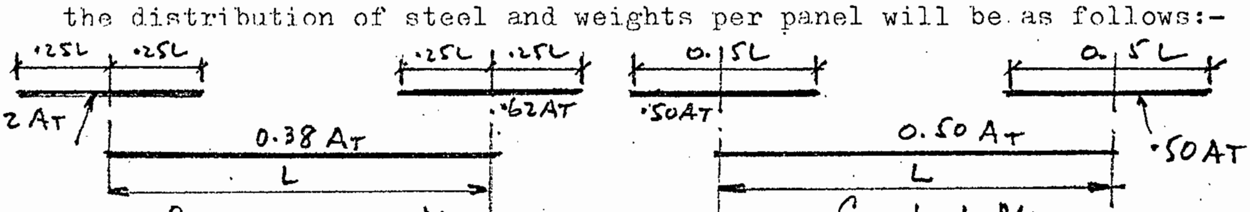


Fig. 26 Recommended Mp | Constant Mp

WT. OF STEEL $\propto$ AT.L (0.38+0.31) | WT. OF STEEL $\propto$ AT.L (0.5+0.25)

Using the recommended values of Mp, a saving of $\frac{75-69}{75} = 8\%$ of reinforcement will result.

4. Although P_2 can be made equal to P_3 by further concentrations of steel over the Column, it is desirable to maintain a difference so that collapse does not occur suddenly, but is preceded by large plastic deformations giving warning of overloads.

5. The above procedure recommended is the same for rectangular panels.

6. For design, external boundary conditions or changes in span are met by simple moment-distribution of the fixed-end moments for the full panel width. The redistributed moments can be distributed across the panel width in the recommended proportions.

7. In the case of slender columns, their stiffnesses may be neglected in (6) above. Where they may not be neglected, their stiffnesses can be included in the moment distribution, but they must be modified by the factors given below owing to the difference in behaviour between a slab to column connection and a beam to column connection. For vertical loading the full slab stiffness is assumed but the column stiffness is multiplied by the factor A.

Ratio $\frac{r_b}{L_2}$	Factor A
0.15	1.23
.125	1.11
.1	1.02
.075	0.91
.050	0.80
.025	0.67
0	0

L_2 is width of the panel normal to the direction of span.

8. The effective column radius allows for the effects of plate thickness and column shape. Square columns may be considered as circular columns of the same area and the effective radius taken as column radius plus half the plate thickness.

F.13. DISTRIBUTION OF MOMENT ACROSS PANEL WIDTH.

In the Appendix, A.F7 – A.F13 which are reproduced
(R20) from Reference 20, show the distributions over the panel widths of the critical positive and critical negative moments, as well as moments along the column centre line, for panel aspect ratios of 1.0, 1.5, 2.0, 3.0. By integration, the average moments in the conventional column and mid-strips of width equal to /

to $\frac{L}{2}$ are shown in tables in the Appendix as follows for an internal panel.

Table A.T1 - As percentages of the Static Moment Mo

Table A.T2 - As percentages of the Critical Moment

Codes of Practice Distribution

For a square panel the actual moment distribution is replaced by the step function shown below in A.C.I. 318. The arbitrary proportions shown are used in the "Empirical Method" and also the "Elastic Method" irrespective of the aspect ratio. The tables above show that the codified step-function may be only applied to square panels as for large aspect ratios the variation is too large.

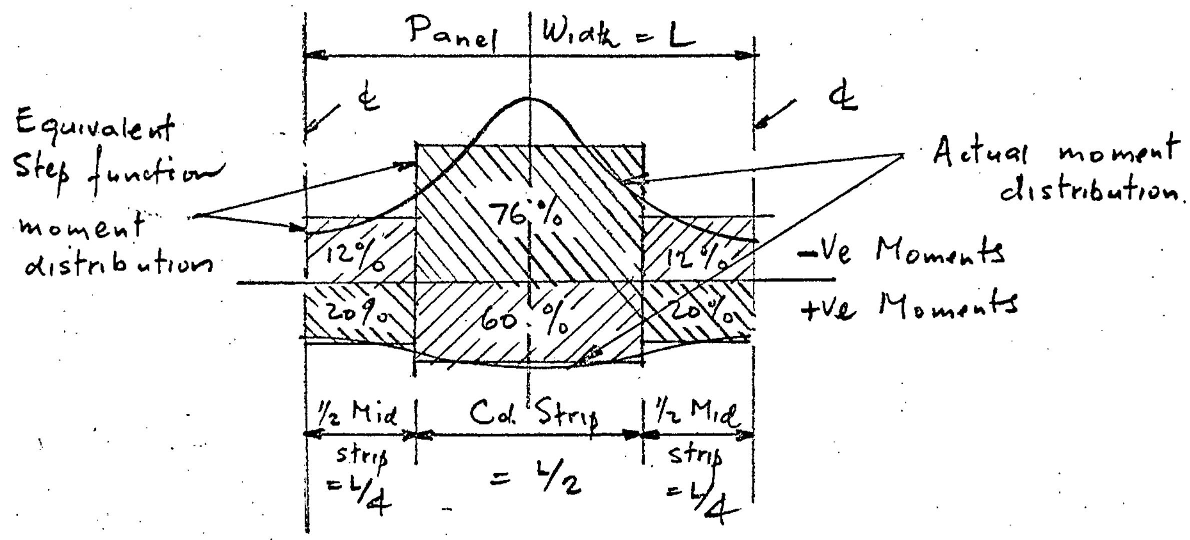


Fig. 27

For example for $\frac{Ly}{Lx}$ = 2 and $\frac{rb}{Ly}$ = 0.05, the distribution of negative moment between the column and the mid-strip is 57 : 43 in the long direction and 91 : 9 in the short compared with the recommended 76 : 24. For positive moments, instead of the 60 : 40 recommended distribution it is 51 : 49 in the long direction and 86 : 14 in the short. However if the panel is partitioned differently so that the width of both column strips is equal to $\frac{Lx \cdot Ly}{Lx+Ly}$ where Lx, Ly are the panel lengths, with the middle strip forming the remainder of the panel width, the codified step-function distribution will closely resemble the actual distribution for aspect ratios from 1 : 1 to 3 : 1.

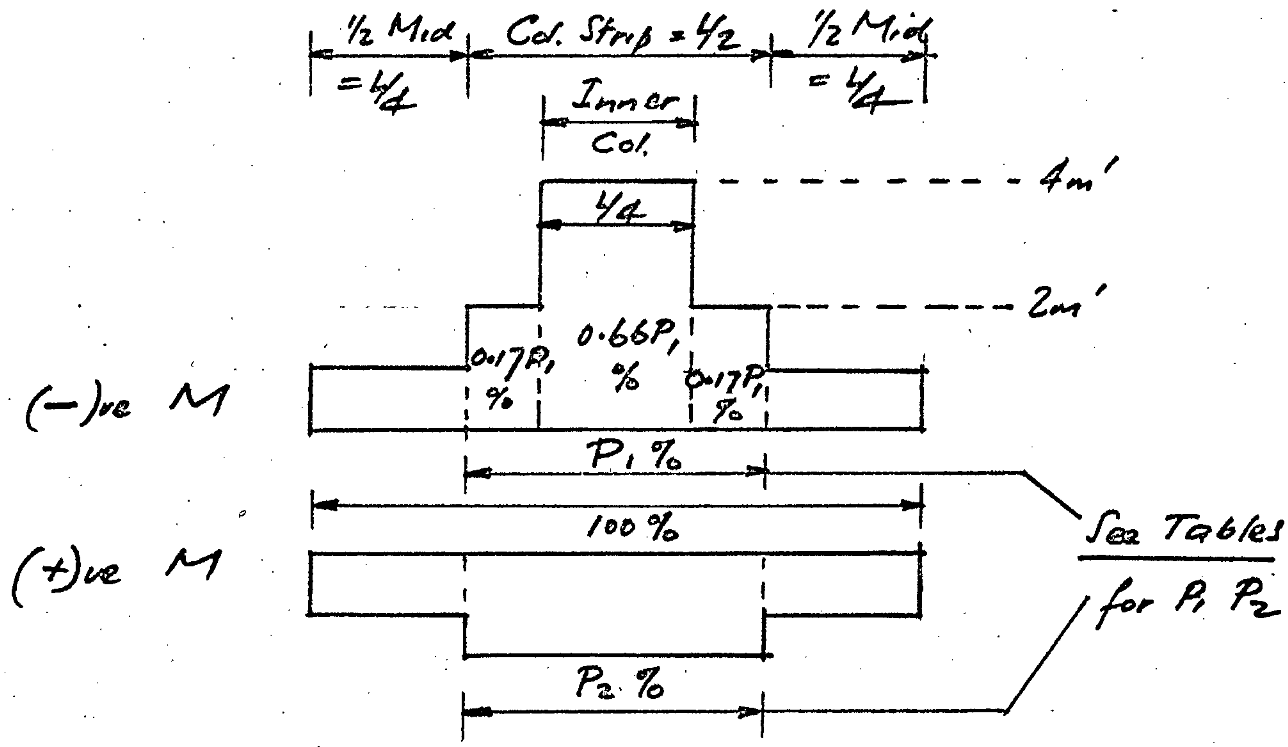
Alternately the conventional division into equal widths may be used provided the division of moments follows the recommendations in Table AT2.

As there is a high concentration of negative moment as the column is approached, it is desirable to subdivide the column

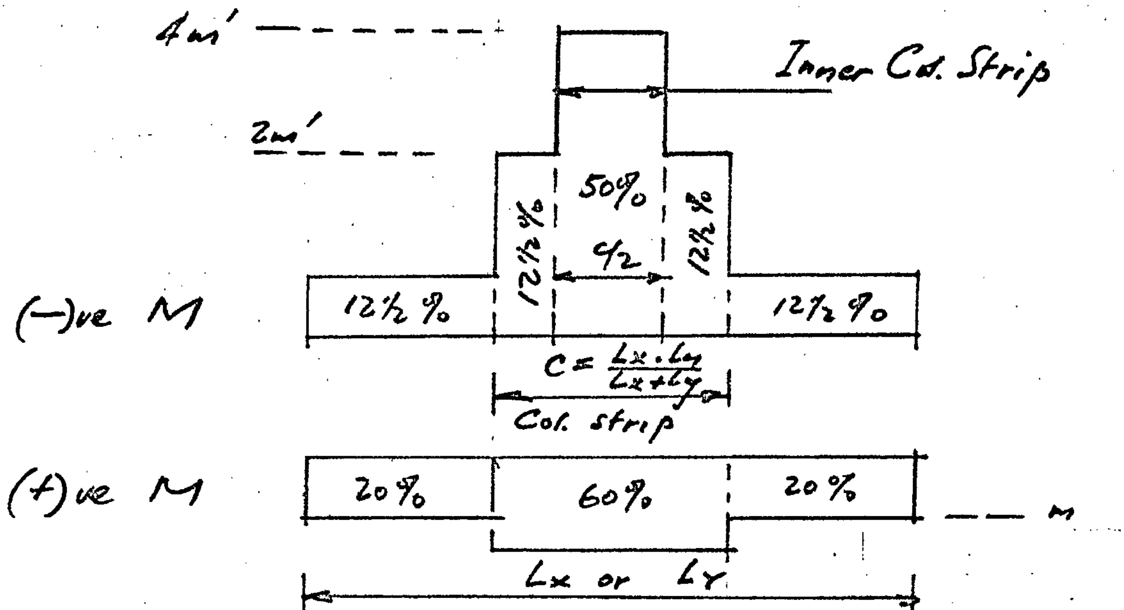
strip /

strip further in to inner section of half the column strip width. The moment in the inner section should be 50% of the total panel moment, if the recommended partitioning is used or 67% of the column strip moment if conventional partitioning into strips of equal width is used.

The two alternative distributions in Fig. 28 are therefore preferred to those recommended in the Codes.



Fig(28(a)) Step-function Moment Distribution for Column Strips of Conventional width = L/2



Fig(28(b)) ALTERNATIVE Moment Distribution for amended Column Strip width $C = Lx . Ly / Lx + Ly$ in both directions

F 2. DIRECT DESIGN FOR OPTIMUM CRITERIA

In analysis, specific values of slab thickness and reinforcement are assumed and the corresponding slab behaviour in terms of stresses and deflections is determined.

The concept of direct design reverses this procedure. A rational and desired behaviour is selected and the slab thickness and reinforcement areas are determined directly to satisfy it. If optimum behaviour i.e. minimum cracking and deflection is required direct design will produce minimum material quantities also. This procedure is applicable to both reinforced and pre-stressed flat plates and is illustrated below by application of the results of the Elastic-Plastic Analysis.

F 2.1. DESIGN PROCEDURE FOR REINFORCED PLATES

1. The plate is divided into strips in each direction along the panel centre lines.
2. Each panel strip is considered as a continuous beam supported at the column centre lines and loaded with its full load.
3. The beam is analyzed by conventional moment-distribution assuming the columns to have zero stiffness. The columns may be included in the analysis with their stiffness modified.
4. The critical negative moment is at the column centre line and is reduced by the quantity Prb/π to allow for column and slab sizes.
5. The resulting moments at the critical sections are distributed across these sections according to Figure 28b or Table A.T2
6. The reinforcement is distributed to resist these moments.

The above procedure will now be utilised to design the previous example of a flat plate in the following Section F.2.2

F2.2 ELASTIC - PLASTIC DESIGN EXAMPLE

The previous example (Fig 16) will now be re-designed directly by this procedure.

Panel size $L_1 = 8000$ $L_2 = 6000$ Thickness = 250
Dead plus Live Load = 13,6 KN/m²

1. Check depth for Shear

$P_{INT} = 13{,}6 \times 6 \times 8 \times 1{,}05^2 = 720$ KN.

$P_{ULT} = 1{\cdot}8 \times 720 = 1300$ KN

$V_{ULT} = \dfrac{P_{ULT}}{b_1 d} = \dfrac{1300}{4\,[500 + (250-50)]\,200} = 2{,}32$ MPa

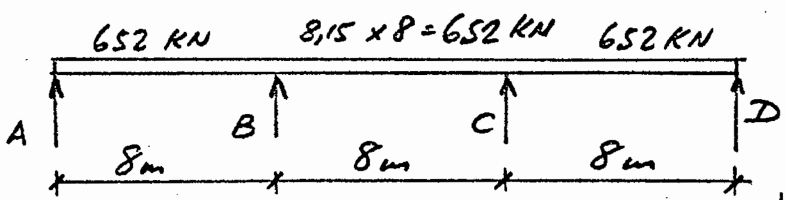
Max $V_{ULT} = 4\sqrt{f_c'}$ p.s.i $= 1{,}52$ MPa (A.C.I)

As previously, retain the same slab thickness but provide Shear-heads as shear reinforcement.

2. Divide into strips in the 8000 direction and analyse for full dead plus live load.

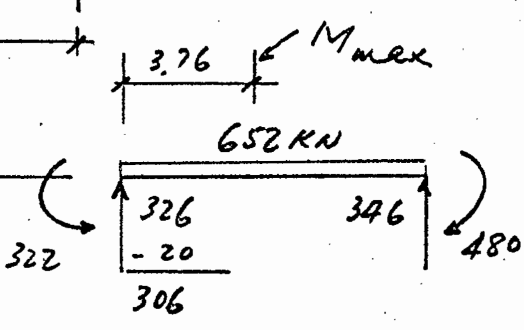
The modified column stiffnesses should be taken into account but as they are only slightly reduced the previous distribution factors will be used.

F. E. Moments $= 652 \times 8/12 = 435$ KN.m.



℄

(·27)	(·24)	(·12)
−435	+435	−435
+118 →	59	0
−7	−14	−7
+2 →	1	0
−322	480	−442



$M_{A/B} = 306 \times \dfrac{3{,}76}{2} - 322 = \underline{253 \text{ KN.m}}$

$M_{B/C} = 326 \times \dfrac{4}{2} - 442 = \underline{210 \text{ KN.m}}$

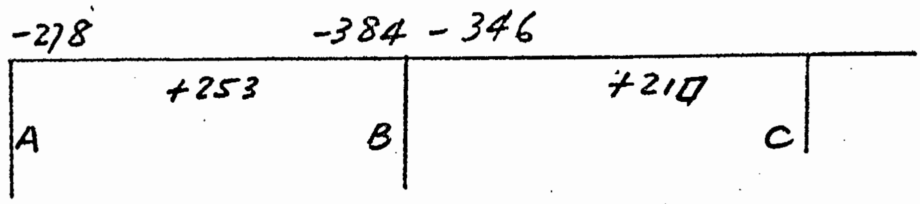
$P_A = 306\ KN. \quad r_b = 0{,}380 \quad M_{AB} = 322 - \dfrac{306 \times 0{,}380}{\pi}$

$= 278\ KN.m \longrightarrow$

$P_B = 346 + 326 = 672\ KN. \quad M_{BA} = 480 - \dfrac{672 \times 0{,}380}{\pi}$

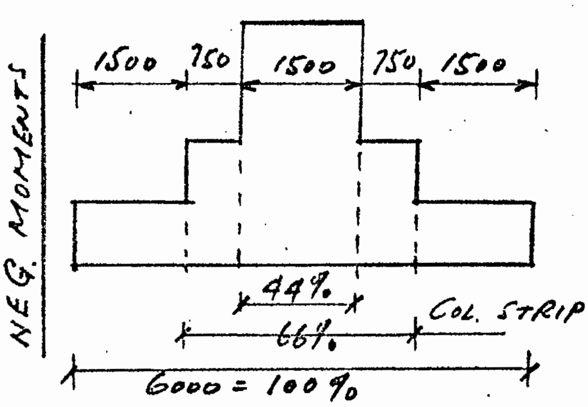
$= 384\ KN.m \longrightarrow$

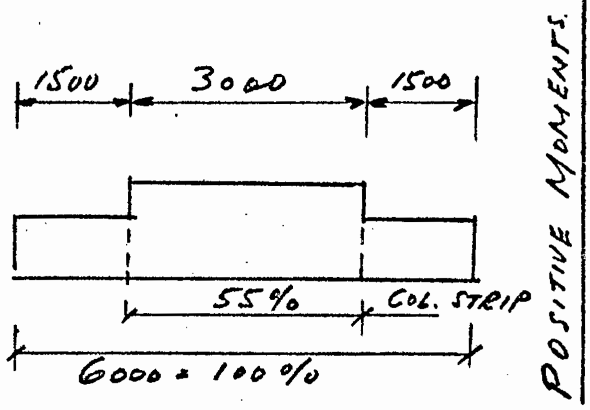
3. Design Moments (M)



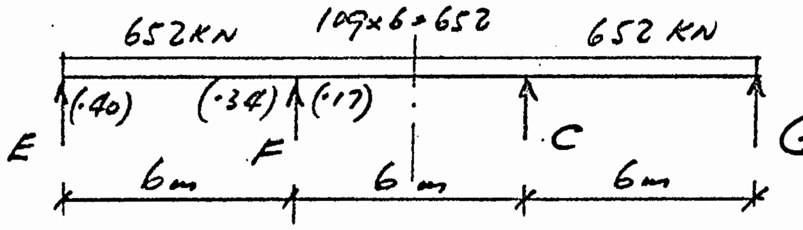
			AB	A/B	BA	B/C	
INNER COL. STRIP	b=1500 d=210	%M	44	28	44	28	%
		m	-122	70	-169	60	KN.m
		As	3200	1800	5000	1500	mm^2
		R	10 Y20	6 Y20	16 Y20	5 Y20	
OUTER COL. STRIP	b=1500 d=210	%M	22	27	22	27	%
		m	-61	70	-85	60	KN.m
		As	1600	1800	2500	1500	mm^2
		R	5 Y20	6 Y20	8 Y20	5 Y20	
MID STRIP	b=3000 d=210	%M	34	45	34	45	%
		m	-95	113	-128	97	KN.m
		As	2400	2900	3300	2500	mm^2
		R	8 Y20	10 Y20	11 Y20	8 Y20	

4. Subdivision into strips according to the following distribution (See Table A)



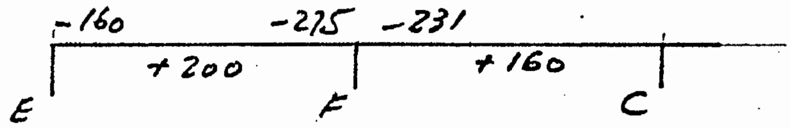


5. Divide into strips in the 6000 direction

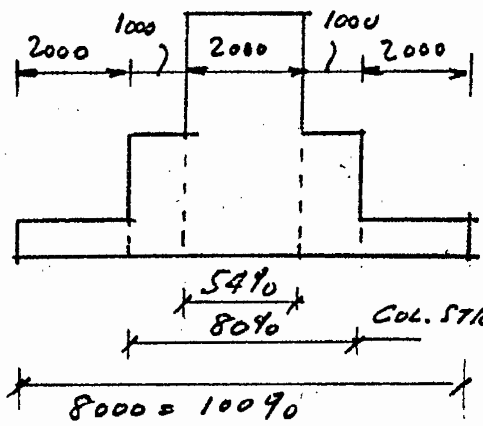


F.Eq. 652×6/12
= 326 KN.m

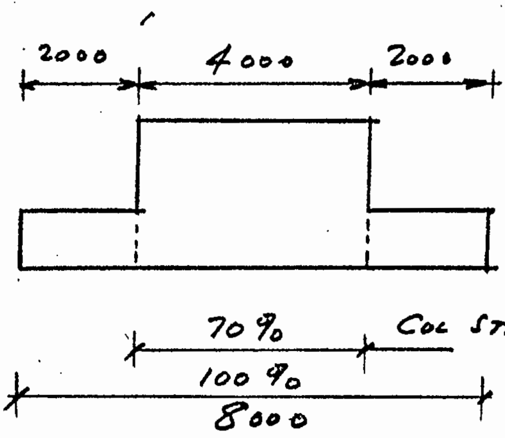
6. Design Moments (M) from Analysis



7. Subdivision into strips according to the following distribution (See Table A)



NEGATIVE MOMENTS



POSITIVE MOMENTS

			EF	E/F	FE	F/C	
INNER COL. STRIP	b=2000 d=190	%M	54	35	54	35	%
		m	86	70	148	56	KN.m
		As	2400	1950	4200	1550	mm^2
		R	12 Y16	10Y16	21 Y16	8Y16	
OUTER COL. STRIP	b=2000 d=190	%M	26	35	26	35	%
		m	42	70	72	56	KN.m
		As	1200	1950	2000	1550	mm^2
		R	6 Y16	10Y16	10 Y16	8Y16	
MID STRIP	b=4000 d=170	%M	20	30	20	30	%
		m	32	60	55	48	KN.m
		As	900	1700	1500	1350	mm^2
		R	5Y16	8Y16	8 Y16	7Y16	

8. Reinforcement Layout

This is similar to the layout in Fig. 19 for the Elastic design EE.1. Only the numbers of reinforcement bars in the column and mid strips need to be altered to satisfy the As requirements in 4) and 7)
From the resulting schedule the weight of the reinforcement is :-

Weight of Panel	$= 3400 \times 3 =$	10 200 Kg
Shear heads	$=$	1 730
TOTAL MASS	$=$	11 930 Kg.

Compare with Total Mass of E1 = 12 000 Kg

9. Check Panel Load for yield at Column edge in all directions.

If M_{px} = yield moment per metre in short direction
M_{py} = " " " " in long "
P = panel load corresponding to yield
L_y : long span $\quad r_b = \frac{c}{\sqrt{\pi}} + \frac{d}{2}$ (average)
L_x : short span
c = side of square col.

$$M_{px} = \frac{P}{4\pi}\left[\log \frac{r_b}{L_x} + 1{,}310\right] \longrightarrow$$

$$M_{py} = \frac{P}{4\pi}\left[\log \frac{r_b}{L_x} + 2{,}310 - \frac{L_y}{L_x}\right] \longrightarrow$$

M_{px} : $\quad p = \dfrac{4200}{2000 \times 190} = 1{,}1\%$, $f_y = 420$, $f_c' = 21$ MPa

From $M_p = p f_y d^2 \left(1 - 0.59 \dfrac{p f_y}{f_c'}\right)$

$$M_{px} = 145 \text{ KN.} = \frac{P}{4\pi}\left[\log \frac{0{,}380}{6000} + 1{,}310\right]$$

$$P = 1250 \text{ KN.}$$

M_{py} : $p = \frac{5000}{1500 \times 210} = 1{,}6\%$ $f_y = 420$ $f_c' = 21$ MPa

$$M_{py} = 240 \text{ KN} = \frac{P}{4\pi}\left[\log\frac{0{,}380}{6000} + 2{,}310 - \frac{8}{6}\right]$$

$$P = 1700 \text{ KN.}$$

The smaller of these two values $P = 1250$ KN corresponds to the load P_2 previously selected as a criterion, so that M_p values at the critical sections are made proportional to the corresponding moments at panel load P_2.

Total Panel Design Load $P_s = 652$ KN

Ultimate Panel Load $= P_s \times$ Load Factor
$= 652 \times 2{,}0$
i.e. $P_{ULT} = 1304$ KN.

Remarks :
The panel will fail at $P_{ULT} \geq 1304$ KN by folding in the short direction L_x due to simultaneous yield at the critical sections. Before $P = 1250$ KN, the behaviour will be elastic with minimum deflection.

10. Check Deflections

From Fig 21 the deflection of the panel centre is given by $\Delta w = \Delta a + \Delta b$ where Δa, Δb can be estimated by the previous method using the column strip in the long direction and the mid-strip in the short. As the new moments are similar in scale to the previous moments, Δw will be slightly less due to the higher negative moment* and reinforcement at the interior support in the long direction, which makes the greatest contribution to the deflection.

F. 3. BEHAVIOUR OF A PRE-STRESSED PLATE

A pre-stressed plate has been found by experiments to behave essentially as an elastic plate provided the ~~t~~endon and concrete stresses remain within their Elastic ranges. This elastic behaviour will continue with increasing load provided that plastic moment or rotation is confined to a line at the column edge.

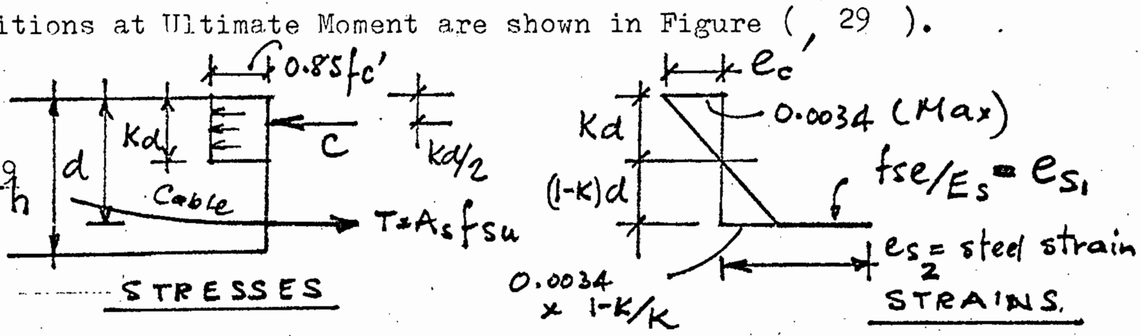
With unbonded tendons, the internal moment reaches its ultimate or plastic value Mp through plasticity in the concrete (at a strain of 0.34%) with or without plasticity of the steel. The expression for Mp is again given by

$$Mp = pfsud^2 \left(1\text{-}059 \frac{pfsu}{d^2}\right) \qquad \text{(A.C.I. 318-63)}$$

where fsu = stress in tendons at maximum moment

Conditions at Ultimate Moment are shown in Figure (29).

Fig.29



In addition to the external transverse loads on the plate and the internal moments produced thereby, the effect of the pre-stress is to superimpose either :-

(a) tendon reactions on the concrete caused by the anchorages, curvatures or changes of slope

or (b) a uniform compression together with internal moments generally of opposite sign to the moments caused by the external loads and equal to Fe_1 (See Figure 30).

The purpose of pre-stressing flat plates is the same as all pre-stressing, namely to reduce internal moments, deflections and tensile stresses throughout. Figure (30) shows the end span of a continuous plate, with cable position and the resulting C line (Centre of Concrete Compression) due to the cable force F and the external Loads. This C line is obtained from the C line due to the cable force by plotting $a = \frac{Mx}{F}$ at every section where

a = shift in C line

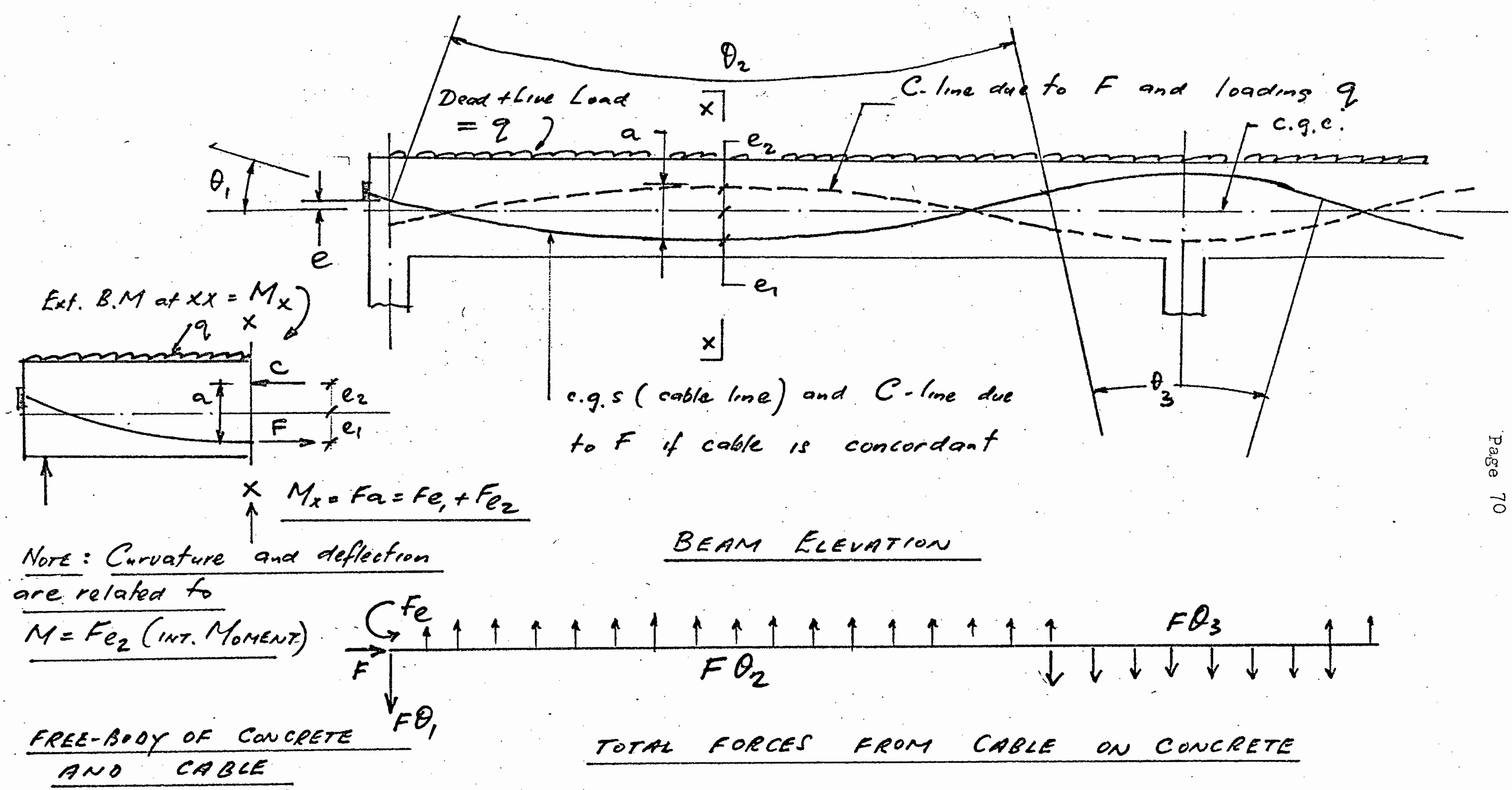
Mx = B.M. due to external loads including dead load

F = pre-stress force

a is above the cable if Mx is positive (sagging)

Alternately the C line can be obtained by an Elastic

analysis/



θ_2
Dead + Live Load = q
C-line due to F and loading q
c.g.c.
θ_1
e
a
e_2
e_1
x
c.g.s (cable line) and C-line due to F if cable is concordant
θ_3
Ext. B.M at XX = M_x
q
C
F
$M_x = Fa = Fe_1 + Fe_2$
BEAM ELEVATION
Note: Curvature and deflection are related to $M = Fe_2$ (Int. Moment)
Fe
$F\theta_1$
$F\theta_2$
$F\theta_3$
FREE-BODY OF CONCRETE AND CABLE
TOTAL FORCES FROM CABLE ON CONCRETE

FIG. 30

analysis of the slab carrying the external loads and the cable forces shown, from which the internal moments M are calculated.

$e_2 = \frac{M}{F}$ is then plotted where

e_2 = distance of the C line from the c.g.c.

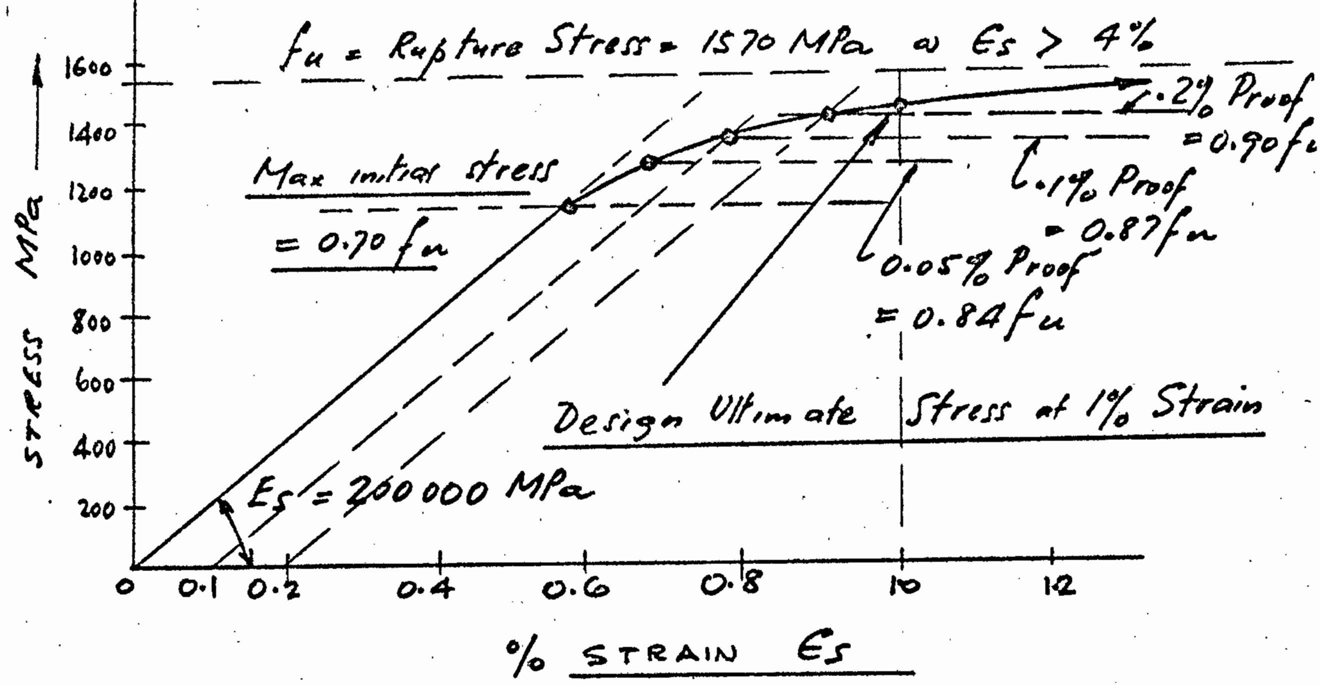


Figure 31 Typical Stress-Strain curve for pre-stressing steel

DEFINITION A Concordant cable produces no external reactions and therefore its compression line coincides with the cable.

NOTE

1. Any real moment diagram by any external load system to scale, is a location for a concordant cable.
2. Any C line is a location for a concordant cable.
3. Linear transformation of a cable does not change the position of the resulting C line.
4. A parabolic cable produces a uniformly distributed reaction ω on the concrete = $\frac{8Fh}{l^2}$ per unit length where h = total drape of the parabola (Fig. 32)

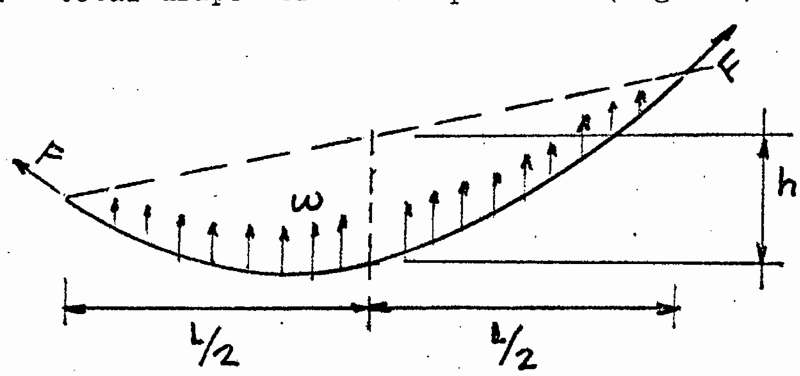


Fig. 32

(R4) By the methods of analysis previously described using the auxiliary elastic foundation, external moments Mx, internal movements M and hence deflections can be calculated as before for any axi-symmetrical loading system. The uniformly distributed loads P1, P2 on an internal panel corresponding to radial yield and tangential yield respectively are related to Mp as before.

Using the load-balancing concept, it is obvious that the cable profile and tension can be directly determined in order to balance the sustained load "qs" exactly. As explained later this is a desirable criterion. At this loading "qs", the internal moments in the concrete are zero and the centre of compression is everywhere at the middle surface.

As the loading increases, moments are produced and the centre of compression is displaced. The maximum moment is the radial moment at the column edge where the centre of compression is furthest from the middle surface. As this moves below the middle third, the top surface moves into tension and later cracking. As the crack progresses downwards, the centre of compression drops to reach an extreme value. The <u>internal moment</u> F x a has reached its constant maximum value Mp at the load P1 allowing plastic rotation of the plate. Thus an effective yield line forms around the column face and the tangential moment increases with increasing load until it too reaches Mp at the load P2. With further load, Mp spreads along the column centre line and then along the panel centre line parallel to it.

When Mp is reached along the entire critical sections, for both positive and negative moment at the load P3, a complete yield line pattern is formed representing the ultimate flexural capacity of the plate.

Hence further desirable criteria would be that $P1 > Pw$ where Pw = working load so that the plastic moment is not reached at working loads, and also that P2 approach P3 ($P2 \rightarrow P3$). By realising these criteria deflection and deformation at service loads and overloads would be a practical minimum. It will now be shown that all these criteria and others can be achieved by direct design so that the behaviour is optimized at all loads and so that the cost of the structure is not affected.

<u>BEHAVIOUR CRITERIA</u> /

BEHAVIOUR CRITERIA

The tendon pattern in an ideal pre-stressed concrete flat plate should satisfy the following criteria :-

1. Ultimate load should be attained with a minimum number of tendons.
2. Deflection and cracking should be minimized at other loads.
3. Deflection should be zero at sustained loads.
4. In plane stresses should be a minimum
5. Tendon profiles should be identical smooth curves.

These criteria are desirable for obvious reasons, but criterion (3) is desirable for several. As the concrete is everywhere in pure direct stress, no bending and no deflection occur. The deformation due to creep is uniform over the depth and produces no deflection either. Hence long term deflection is a minimum. The slab is also crack-free and water-tight, brick partitions are crack-free, the elastic range for lateral or dynamic loading is a maximum and overall maintenance of the structure is minimized.

A solution is possible by the following procedure which satisfies all these criteria very nearly.

F. 3.1. DESIGN PROCEDURE

1. The plate is divided into panel strips in each direction
2. The panel strip is considered as a continuous beam supported at the column centre lines.
3. The plate thickness is determined from the ultimate shear or from the deflection under full live load.
4. The beam is analyzed by conventional moment distribution carrying the full load assuming the columns to have zero stiffness. This assumption is allowable since at zero deflection no moments will be transferred to the columns. Negative moments are reduced by $\frac{Pr_b}{\pi}$ to allow for column and slab size.
5. The distance "e" of the cable from the middle surface is made proportional to these adjusted moments M. At the columns, "e" has the maximum value of half plate thickness less the required cover, and the cable is rounded

over /

over a width from 0.3L - 0.4L to reduce friction losses.

6. The total number "N" of cables per panel width is determined from the Ultimate panel load and the corresponding numerical sum of the total ultimate moments at the critical sections.

7. The number "n" of cables per unit width is made proportional to the distribution of critical negative moments across the panel width at the load P2. This distribution varies with panel shape and is approximated by the bands shown in Figure (34). For ratios of $\frac{L}{S}$ greater than 2 : 1, adopt the latter distribution.

8. The effective cable tension To (after losses) is determined to balance the external moments at the sustained loading and is evaluated at the panel centre giving

$$To = \frac{M}{Ne}$$

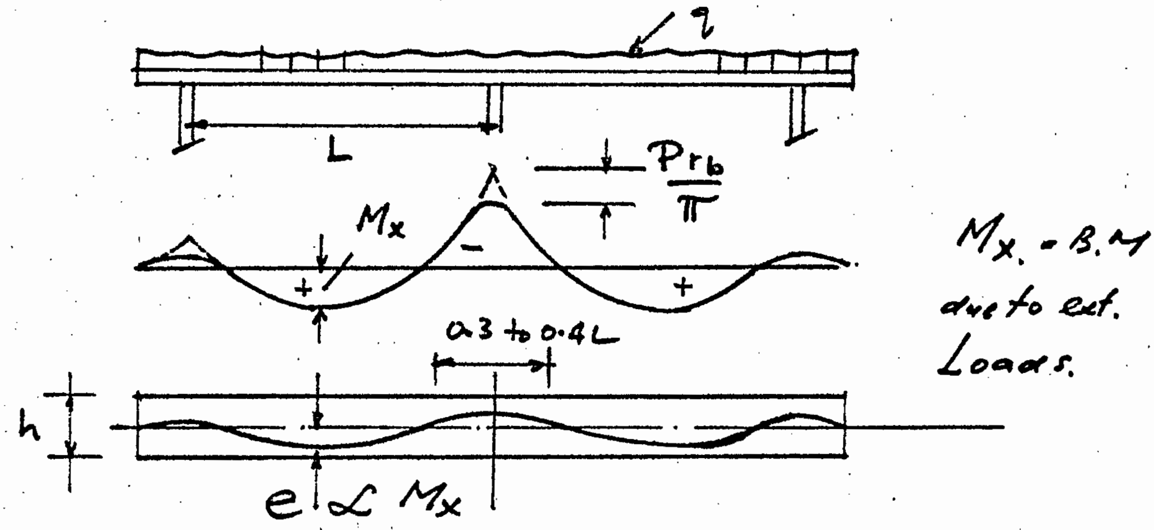


Fig 33 Loading, Moments, Cable Profile

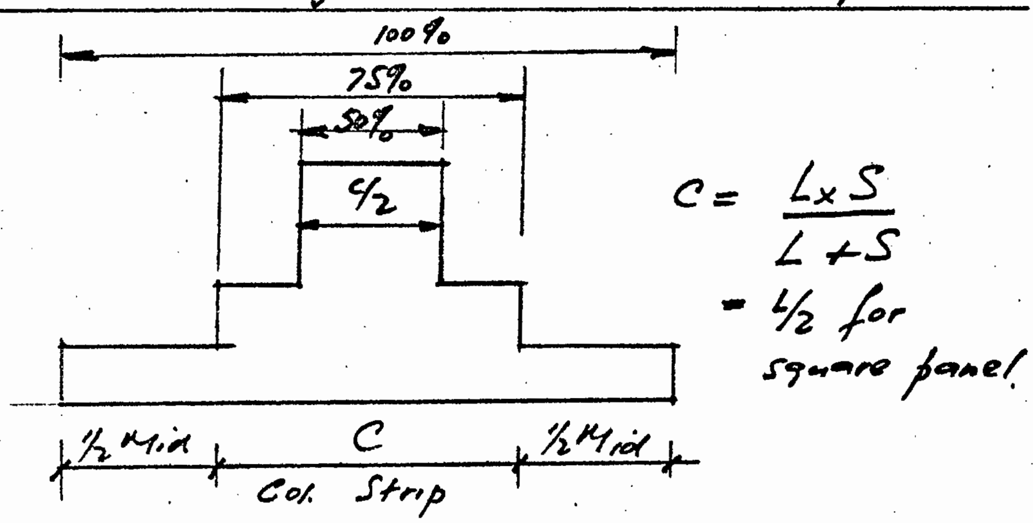


Fig. 34 Optimum Cable Distribution in Rectangular Panel

REMARKS

1. Shear Strength

Little consideration has been given to the shear strength of pre-stressed slabs beyond a limited series of tests by Scordelis, Lin and May from which the following Empirical equation was derived :-

(R32) $$\frac{V_u}{bd} = f_c^1 \left(0.175 - 0.0000242\, f_c^1 + 0.000020\, \frac{F_e}{S}\right) \text{p.s.i.} \quad \ldots\ldots (57)$$

where V = ultimate punching shear load (lbs)

b = perimeter of the loaded area

d = effective depth of slab

F_e = effective total pre-stress in each cable prior to loading (lbs)

S = spacing of cables (ins.) in the zone of critical shear

Although logically speaking, pre-stressing should increase the shear strength and estimates vary from 0 to 30%, it would be advisable to ignore the extra strength until more exhaustive tests can be carried out. At present therefore the formulae for reinforced concrete slabs recommended by Moe, Tasker, Whitney or the A.C.I. in Section (c) should be used.

2. Cable Profile

From step (5) in the design procedure, "e" will not have its maximum value near the mid-span section of the external spans. If the maximum "e" is used at this section as well as the first internal support, the total number of cables can be reduced from step (6) by ≈ 20% as maximum cable eccentricities are employed. This can be seen from figure (35). The respective eccentricities near mid-span are h and $e = J/2 < h$ and for the same size of cable, the ultimate loads are in the ratio of $2\frac{1}{2}h : 2h$ approximately.

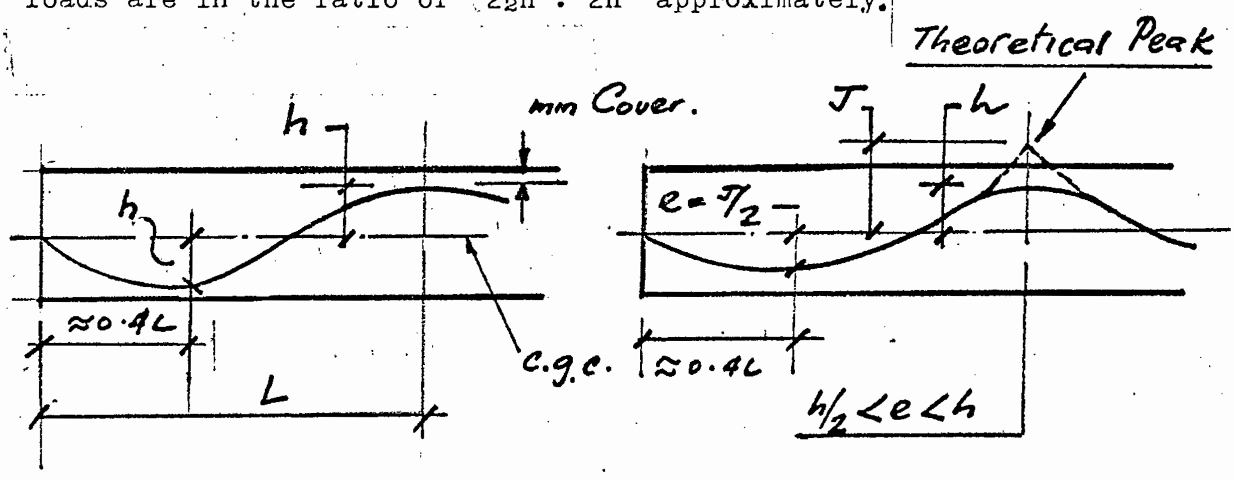


Fig. 35 (a) Writer's Amended Eccentricity (b) Ref. 4 Recommended Eccentricity

Under sustained load, moments near mid-span can be exactly balanced from step (8). However due to the rounding off of the cables for a short width ($\pm$ 0.35L) over the columns, the effective drape of the parabolic profile of the cables between their ends and the commencement of the rounding off, will be reduced so that the net upward reaction produced by the cables is also reduced. (Alternately, the cable moments are reduced about the c.g.c. thereby reducing the uplift). Furthermore, the rounding off of the cables transfers and distributes the entire sustained panel load to a small square over the column of side $\pm$ 0.35L. Due to the aforementioned reasons the panel deflections will be slightly increased but may be compensated by a small increase in the number of cables.

(R32) Consequently the writer considers that the use of the maximum eccentricities is justified in view of the overall reduction produced in the total number of cables required.

3. Tendon Distribution

A Tendon pattern may be also simply determined by beam and slab analogy. Assume imaginary beams between the columns of stiffness equal to the plate stiffness. The slab between beams is balanced by uniformly spaced cables between beams each set carrying one-half of the panel load P and transferring their load to the beams. Thus each beam carries one quarter of the panel load on either side and requires the same number of cables as the full slab panel spanning in the same direction i.e. 50% each of the total cables for any panel. If the beam is assumed to have a width equal to the column strip and its cables are spread over this width in addition to those required for the slab, the following distribution will result in a square panel.

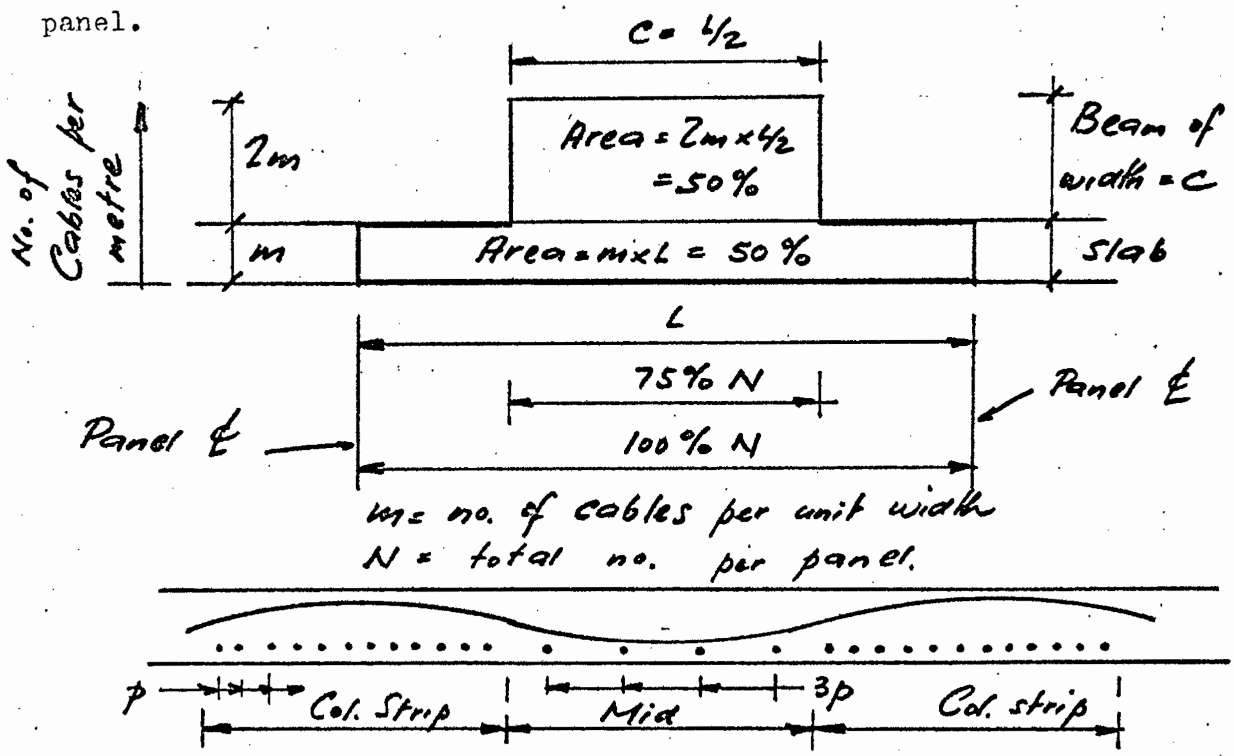


Fig. 36 Cable pattern from analogy to two-way beam and slab

This distribution is identical to that recommended for square panels, and can be applied to rectangular panels using the width of column strip determined from

$$C = \frac{L \times S}{L + S} \quad \text{....................(58)}$$

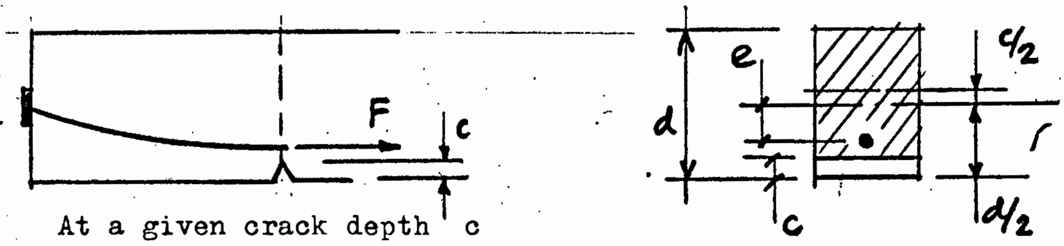
4. Sudden Collapse after Cracking

The cracking moment of a pre-stressed section is the moment at which f_t = maximum tensile stress = modulus of rupture (f_r)

$$f_r \text{ is given approx. by } \frac{3000}{3 + \frac{12000}{fc^1}} \text{ p.s.i. .....(59)}$$

Cracking will continue if the tensile stress at the top of the crack is greater than f_r. In slabs or rectangular beams with unbonded tendons, F = pre-stress does not increase significantly and is assumed constant.

Fig. 37



At a given crack depth c

(R40)

$$M = \left[bf_r + \frac{F}{d-c} \right] \frac{(d-c)^2}{6} + F \left(e + \frac{c}{2}\right) \text{ ....(60)}$$

$$\text{Slope } \frac{dM}{dc} = - \frac{1}{3} \left[b \ (d-c) f_r - F \right]$$

for $c = o$, $\frac{dM}{dc} = - \frac{1}{3} \left[bdf_r - F \right] \geq 0$ for stability at cracking

If $F < bdf_r$, $\frac{dM}{dc}$ is < 0 and as soon as the cracking moment is reached, the section continues to crack and becomes unstable, leading to sudden collapse. The theory has been experimentally verified.

Hence as the tendons in slabs are not usually bonded for economy, it is necessary for the average prestress $\frac{F}{bd}$ to be greater than f_r. This requirement will safeguard slabs against sudden collapse due to overloads or earthquakes.

5. Effect of Rounding Profile over the Column

From the analogy with the two way slab and beam, it was shown that the sustained loads could be balanced everywhere by the cable reactions except in the region of the columns.

In /

In this region the cables have negative curvature and the load is transferred from the cables to the slab. If this transfer is axisymmetric about the column and is concentrated at a line of effective radius r_t, the concrete slab is acted upon by 2 ring loads of magnitude P, upward at radius r_b and downward at radius r_t.

The deflection at the panel centre

(R.4) is $\doteqdot$ $0.019 \frac{P}{D} (r_t^2 - r_b^2)$ ....(61)

The ratio of this deflection to the deflection under uniform load and zero pre-stress is

$$3 \cdot 6 \,(r_t^2 - r_b^2) \; L^2$$

Hence To should be increased by this proportion which for $\frac{rb}{L} = 0.05$, $rt/_L = 0.125$ is a correction of 5%

6. Effect of Partial Pre-stressing

This can be achieved by the addition of non-prestressed reinforcement throughout the bottom of the slab and over the interior columns, being designed for the Ultimate Load at which they will be fully effective.

However, the non-prestressed reinforcement will not retard the onset of cracking but once this has occurred, it will reduce the severity of cracking and increase the resilience and ability to absorb impact loads.

Where the ratio of live load to dead load is high say more than 1•5:1 and where only a small proportion of the live load will be sustained for long periods, its use is recommended as a replacement for some prestressing steel, but further research is necessary.

For all other flat plates, a small amount of non-stressed reinforcement is recommended say 0.15% of the gross area of the slab in each direction, as a supplement to the prestressed tendons.

EXAMPLE

The previous example will be designed as a prestressed plate using the procedure outlined.

F 3. 2. DESIGN OF PRESTRESSED PLATE

Panel Size : $L = 8000$ $S = 6000$ (Fig 16)
Select slab thickness $t = \frac{L}{40} = 200$ mm.

Dead Load :	Slab	2400×200	$= 4,7$ kN/m^2
	Finish		0,5
Live Load			7,2
	Total Load		$= 12,4$ " "

1. Check depth for Shear $f_c' = 3400$ psi $= 24$ MPa
$u_w = 30$ MPa

Modulus of Rupture $f_r = 8\sqrt{f_c'} = 470$ psi

or from Eqtn (59) $f_r = \dfrac{3400}{3 + \dfrac{12000}{3400}} = 520$ p.s.i

(R40) Provide min. average prestress = 500 p.s.i say
$P_{INT} = 12,4 \times 6 \times 8 \times 1,05^2 = 660$ kN. $\longrightarrow$
From Eqtn (57)

(R32) $$V_u = f_c' bd \left[0.175 - 0.0000242 f_c' + 0.0002 \frac{F_e}{S}\right]$$

$$= 24 \times 4 \times 500 \times 150 \left[\begin{array}{l} 0.175 - 0.0000242 \times 3400 \\ + 0.0002 \times 500 \times 8 \end{array}\right]$$

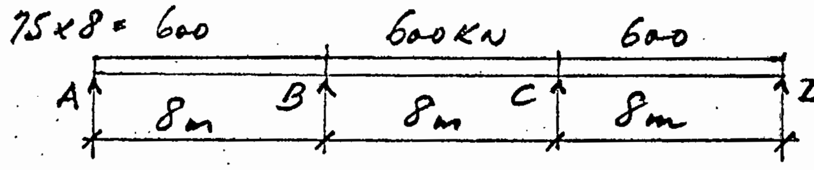
$$= 7200 \times 0.157$$

$$= 1260 \text{ kN.} \longrightarrow$$

Factor of Safety against Shear failure
$= 1260/660 = 1.9$ which is satisfactory but will be checked again when the cable forces have been determined.

(R32) NOTE : If the slab is not pre-stressed ie $F_e = 0$ the above shear formula by Lin gives $V_u = 670$ kN compared to the A.C.I formula $V_u = 4\sqrt{f_c'}$ which gives 630 kN.

2. Divide into Strips in the 8000 direction and analyse for full dead load plus live load. The columns are assumed to have zero stiffness and the negative moments will be reduced by $\frac{P r_b}{\pi}$ F. EM = $600 \times 8/_{12}$ = 400 KN.m

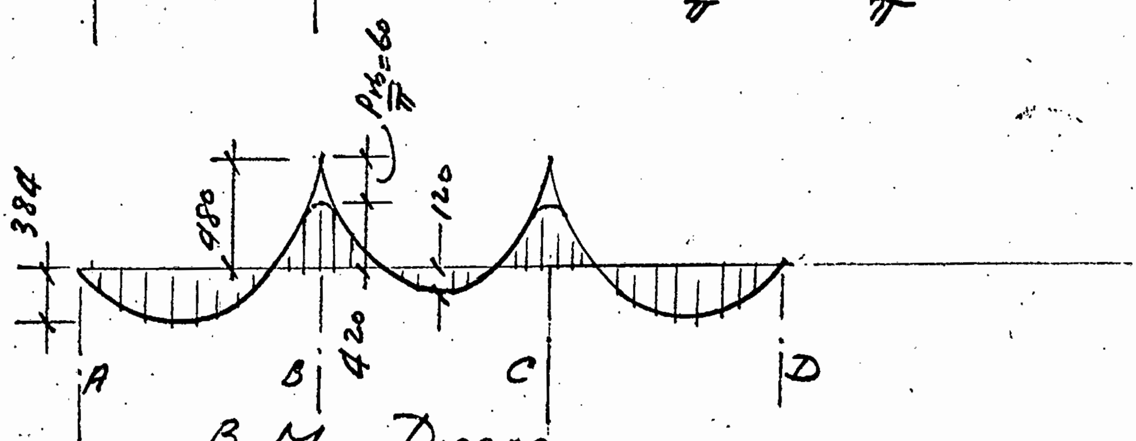


	.60	.40
-400	400	-400
400 →	200	
	-120	-80
0	480	-480

$P = 600$ KN

$r_b = 280 + \frac{150}{2} = 355$ mm

$\frac{P r_b}{\pi} = \frac{600 \times .355}{\pi} \approx 60$ KN.m.



B. M. Diagram

3. Cable Profile

The cable must be draped so that the eccentricity "e" below the middle surface is everywhere proportional to the external moments analysed above.
Hence the cable is very nearly concordant.
At B, C, allow the max eccentricity e = 100 - 35 = 65 mm
and round off the peaks to a width of
≈ 0.3L = 2,500 m.
At the section for max positive moment in AB
$e = 65 \times \frac{384}{420} = 60$ mm ⟶

At mid-span of BC, $e = 65 \times \frac{120}{420} = 20$ mm ⟶

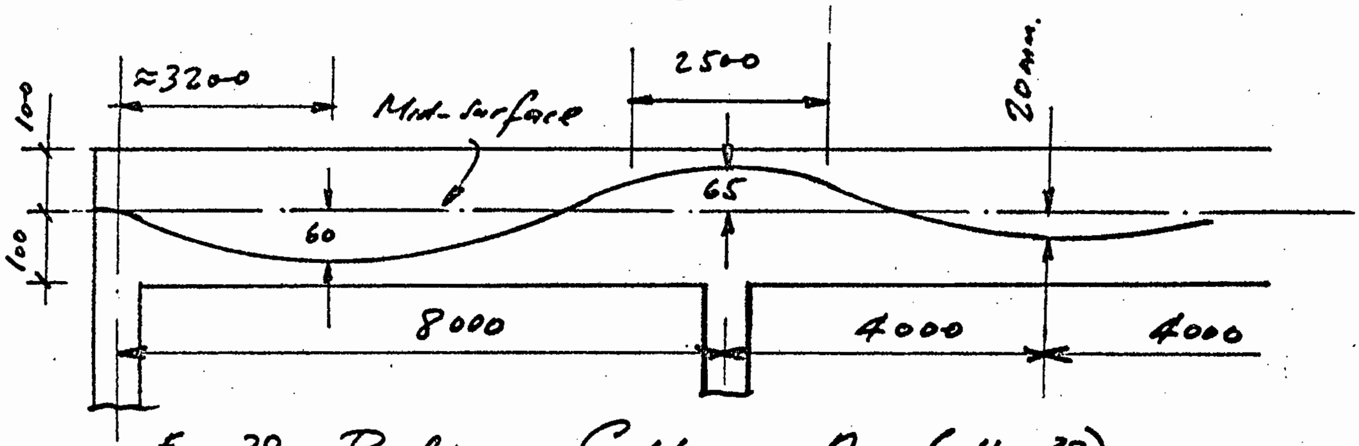


Fig 38. Profile Cables A (N = 32)

A. Total Number of Cables.

Ultimate Load with over-all Load Factor = 1.8

$P_{ULT} = 1.8\ (600) = 1080\ KN.$

Total Panel Moment at Ult. Load

$$M_o = P\left[\frac{L}{8} - \frac{r_b}{\pi}\right] = 1080\left[\frac{8}{8} - \frac{0.355}{\pi}\right] = 960\ KNm \longrightarrow$$

Use 15 mm (0.6") diam. 7 wire strand unbonded cables. Ultimate strength = 250 KN.

After losses effective force $p_e.A_s = 0.55 \times 250$

$= 137\ KN. \longrightarrow$

At Ultimate Load cable force $f_m.A_s$

$\approx 1.25 \times 137 = 171\ KN.$ say. $\longrightarrow$

From C.P. 115 - 313

Ultimate Moment $M_u = N.f_m A_s(d_1 - 0.4d_n)$

where N = no. of cables. $d_n = 0.5\,d_1$

End Span:

Mid-span: $M_{u_1} = \frac{N \times 171}{1000}\left(160 - 0.4 \times \frac{160}{2}\right) = 22N$

Support Col. B: $M_{u_2} = \frac{N \times 171}{1000}\left(165 - 0.4 \times \frac{165}{2}\right) = 22.5N$

$M_o = M_{u_1} + \frac{1}{2} M_{u_2} = 33.3N = 960\ KN.m.$

Approx $N = 29 = \underline{32 \text{ Cables}}$ say.

Check Centre Span:

Mid Span: $\frac{N.p_e A_s}{u_w b d_1} = \frac{137 \times 32}{30 \times 6 \times 120} = 0.2 \quad \frac{f_m}{p_e} = 1.2.$

Table 8 $\quad M_{u_1} = 32 \times 1.2 \times 137\ (120 - .24 \times 120)$

$= 600\ KN.m$

$M_{u_2} = 32 \times 1.3 \times 137\ (165 - .20 \times 165)$

$= 750\ KN.m.$

$M_o = M_{u_1} + M_{u_2} = 600 + 750 = 1350 > 960$ kN.m

Check End Span

Mid Span $\quad \dfrac{N p_e A_s}{u_w b d_1} = \dfrac{32 \times 137}{30 \times 6 \times 160} = 0.153 \quad \dfrac{f_{ps}}{f_{pe}} = 1.3$

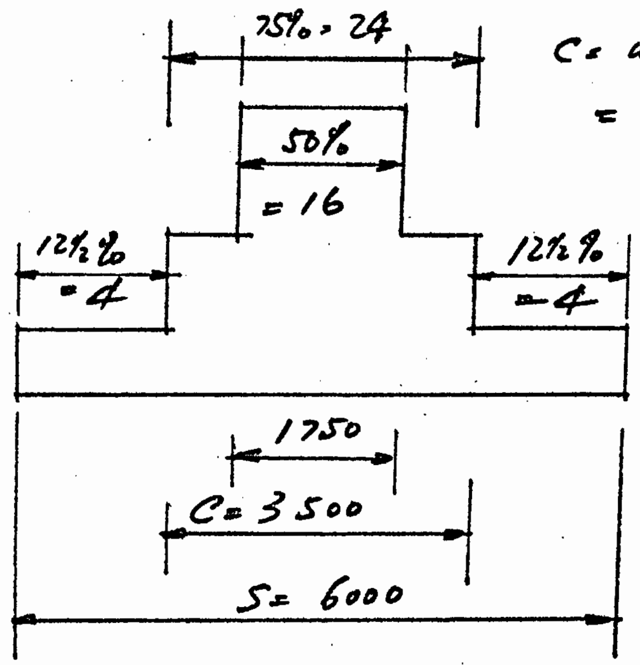
$M_{u_1} = 32 \times 1.3 \times 137 \, (160 - 0.20 \times 160)$
$= 730$ KN.m.

$M_o = M_{u_1} + \frac{1}{2} M_{u_2} = 730 + \frac{1}{2} \times 750$
$= 1125$ KN.m > 960 KN.m.

5. Number of cables "n" per unit length

This is proportioned as follows :-

$N = 32$



$C =$ width of Col strip $= \dfrac{L \times S}{L + S} = \dfrac{8 \times 6}{8 + 6}$
$= 3.5$ m

6. Effective Tension T_o

This is determined to balance the sustained load (load-balancing concept) or the moments produced by them (moment-balancing concept). Both concepts produce a C-line which coincides with the middle surface i.e. zero moments and deflections throughout.

Dead Load = 5,2 KN/m²
Sustained Load = 1/3 LL 2,4 "

Total Sustained Load 7,6 KN/m²

End Span : Max positive moment at sustained load
$= 384 \times 7{,}6/_{12,4} = 235$ KN. m →

by proportion from the moment distribution.

Effective $T_o = \dfrac{M_s}{N \times e_c}$ where

M_s = moment at span centre at sustained load

N = no. of cables and

e_c = cable eccentricity at span centre

$T_o = \dfrac{235}{32 \times 0.060} = 133$ KN.

Add losses $= {}^{15}/_{55}\ T_o = 37$ KN.

Initial Cable force = 170 KN. →

This equals ${}^{170}/_{250}$ = 68% of the Ult. Strength

7. Repeat Steps 2) to 6) in the 6000 Direction

The number of 15 mm ϕ unbonded cables required is 30 with an initial tension of 185 KN. = 74% of the Ult. Strength.

The cable profile is shewn below :-

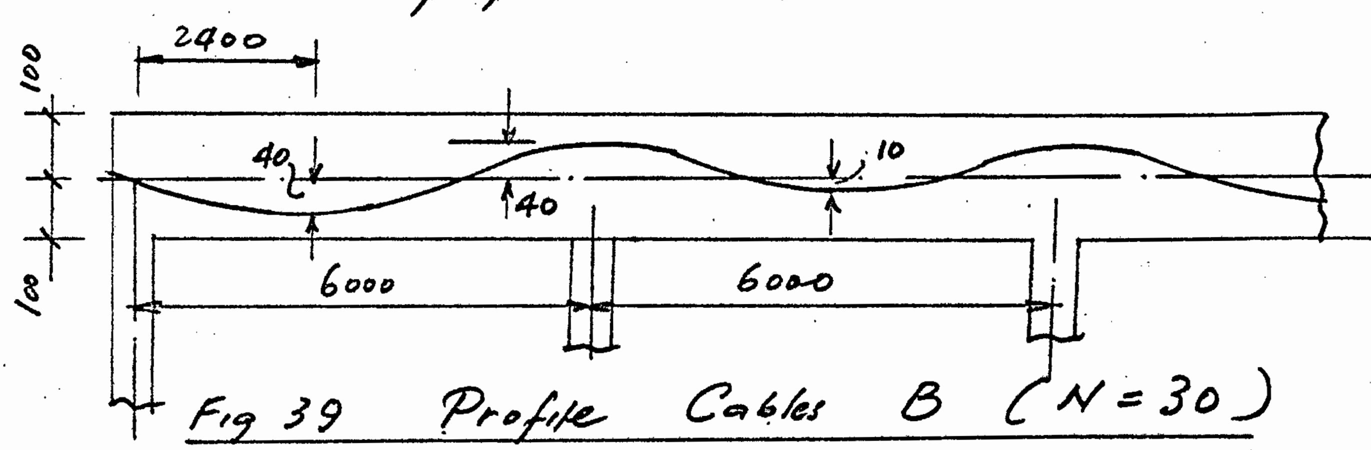


Fig 39 Profile Cables B (N = 30)

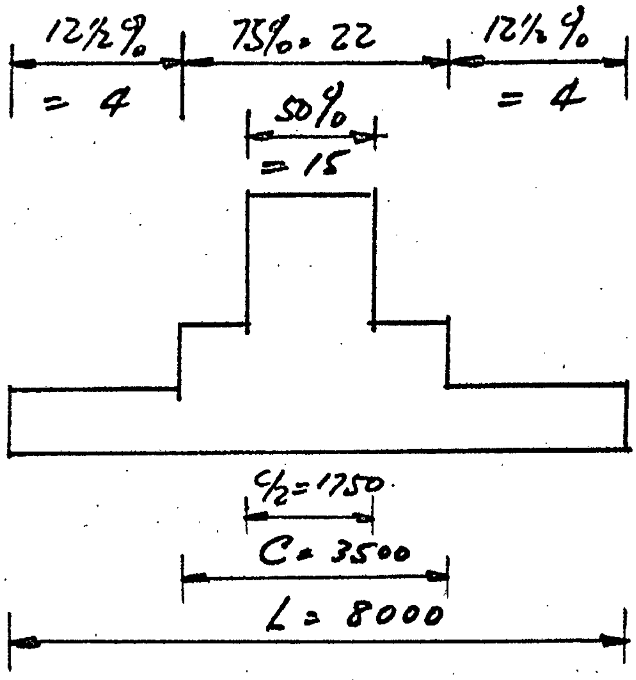


Fig 40

N = 30

8. The Number of cables "n" per unit length.

This is proportioned as above. (Fig 40)
The total no. of cables for Ultimate Load = 24, but this has to be increased to 30 so that the effective Tension T_0 after losses will be 55% to 60% of the Ultimate cable strength. This is checked as follows :-

9. Effective Tension T_0

End Span : Max positive moment at sustained Load of 7.6 KN/m^2

By proportion $M_S = \frac{7.6}{12.4} \times 288\ KN.m = 177\ KN.m. \longrightarrow$

$e_c = 0.040$
$N = 30$

$T_0 = \frac{M_S}{N \times e_c} = \frac{177}{30 \times 0.040} = 148\ KN$

$= \frac{148}{250} = \underline{58.4\%}$ Ult Strength

Add losses $= \frac{15}{55} T_0 \approx \underline{37\ KN}$

Initial Cable force $\approx \underline{185\ KN} \longrightarrow$

10. Cable Layout

The final layout is shown in Fig 41.
The total cost of Concrete and Cables is $\underline{R11\text{-}50/m^2}$

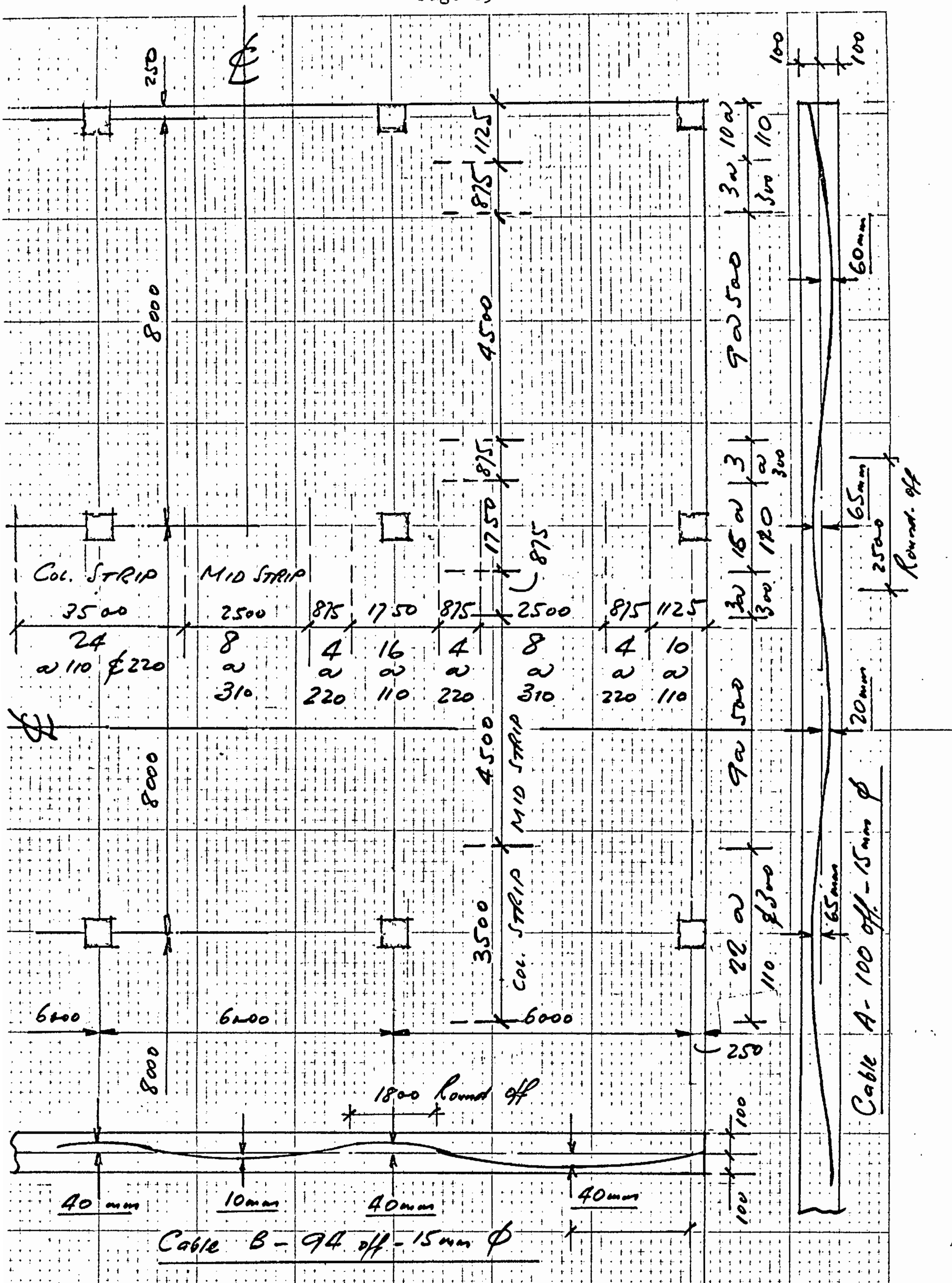


Fig. 41 CABLE LAYOUT - 200 mm PRESTRESSED PLATE F.3.2.

L. LOAD = 7,2 kN/m²

G. FINITE DIFFERENCE METHOD

G. 1. The Elastic Method for the analysis and design of rectangular flat plates described in Section E.1, assumes that the slab may be subdivided along the panel centre lines where shears are zero by symmetry, into frames consisting of strips of slab supported by columns and that each such frame can be analysed independently to determine the total elastic moment at any section. Tests on plates have shown that these assumptions are reasonably valid. However the variation of moment along the chosen section cannot be determined by this method and so step functions are chosen only for the critical sections to represent the integration of the moment curve. In the Elastic-Plastic Method described in Section F, the step functions can be sketched by means of a simple formula and rule or the use of Tables (See Appendix) to represent the moment distribution at prescribed boundary conditions. The areas of reinforcement are then calculated and result in 2 reinforcement ratios at mid-span and 3 ratios at the supports.

To achieve the most economical and safest distribution of reinforcement however, more variations are required in the reinforcement ratio corresponding to the actual variation in the moments. The exact unit moment at any point can only be determined by the Elastic Theory described in Section 2.A. using Fourier Series and this is known as the "Classical Approach". Difficult boundary conditions have prevented the tabulation of solutions for all but simple standard cases such as square or circular simply supported slabs.

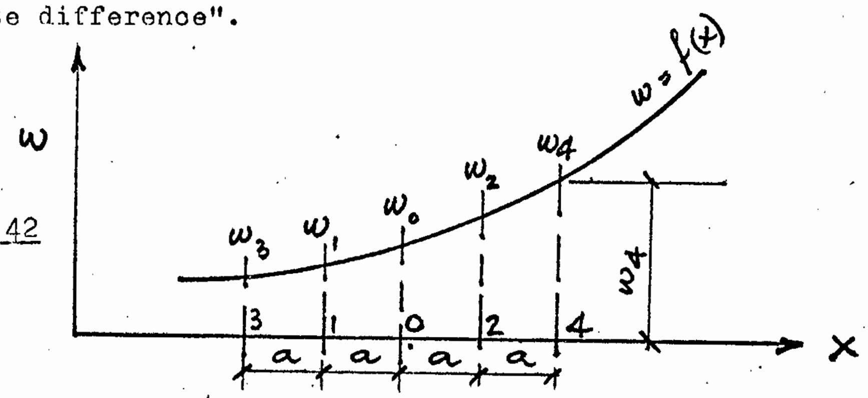
Fortunately the differential equations

$\frac{\partial w}{\partial x}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^3 w}{\partial x^3}, \frac{\partial^4 w}{\partial x^4}, \frac{\partial^4 w}{\partial x^2 \partial y^2}$ etc. at any point can be approximately expressed in terms of the values of "w" (deflection) at the point and at the neighbouring points. For example in Figure 42

$$\left(\frac{\partial^4 w}{\partial x^4}\right)_0 = \frac{1}{a^4}\left(w_3 + w_4 - 4w_1 - 4w_2 + 6w_0\right)$$

where "a" is an arbitrary but small parameter termed a "finite difference".

Fig. 42



Any rectangular slab may be then arbitrarily sub-divided into a square or rectangular mesh and the intersections thereof (nodal points) numbered. The governing biharmonic equation

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad \text{........(62)}$$

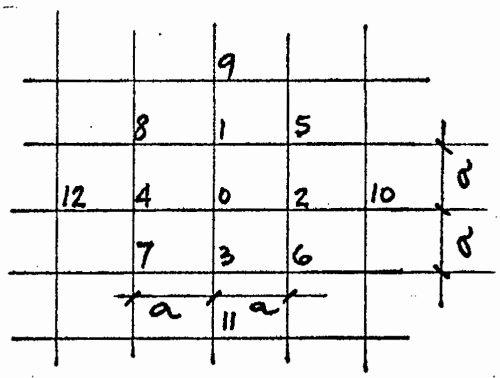
at any nodal point is then expressed in terms of the deflections at surrounding points, the mesh size and the average loading at the point.

For example from Figure 43

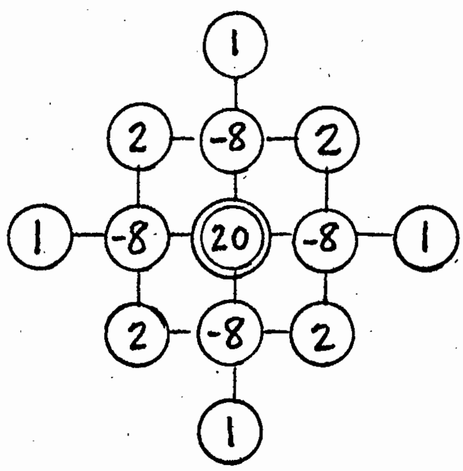
$$\left(\nabla^4 w\right)_o = \frac{1}{a^4}\left[\begin{array}{l} 20w_o - 8\left(w_1+w_2+w_3+w_4\right) + 2\left(w_5+w_6+w_7+w_8\right) \\ +\left(w_9+w_{10}+w_{11}+w_{12}\right)\end{array}\right] \quad (63)$$

where w_o - w_{12} are the values of w at points o - 12 shown

Fig. 43



In operator form, the general operator on the deflections is



$$\approx \frac{a^4 q}{D} \quad \text{.....(64)}$$

At points on or close to the slab boundaries the general operator can be adjusted provided at least two boundary conditions are known e.g. a simply supported edge has

$$\frac{\partial^2 w}{\partial x^2} = 0 \;, \quad w = 0$$

and

a clamped edge has

etc.

$$\frac{\partial w}{\partial x} = 0 \;, \quad w = 0$$

In /

In this way the biharmonic equation at each nodal point can be replaced by a finite difference approximation. As many simultaneous equations can be set up as there are unknown deflections. These are represented in Matrix Form by

$$\left[\nabla\right] \cdot \left[w\right] = \left[L\right] \cdot \left|\frac{qa^4}{D}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

$\left[\nabla\right]$ is a square matrix n x n

where n = number of <u>unknown</u> deflections and row "J" represents the operator pattern at nodal point "J".

$\left[W\right]$ is a column matrix 1 x n representing the unknown deflections

$\left[L\right]$ a column matrix 1 x n representing one of any number of loading patterns

$\left|\frac{qa^4}{D}\right|$ is a constant representing the maximum loading at any point and the slab and mesh parameters.

The "n" equations are rapidly solved by electronic computer for each loading pattern. Having obtained the deflections at all the nodal points in each case, the unit values of Mx, My, Mxy etc. are then calculated at each point by finite difference approximation using a different computer program. For example

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \approx \begin{array}{ccc} & \nu & \\ 1 & -2-2\nu & 1 \\ & \nu & \end{array} \times -\frac{D}{a^2}$$

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \approx \begin{array}{ccc} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{array} \times \frac{D(1-\nu)}{4a^2}$$

where ν = Poissons Ratio

$D = \dfrac{Eh^3}{12(1-\nu^2)}$ = Flexural Rigidity of the Plate

FURTHER ADVANTAGES

1. Provided D is constant throughout, the Moments are independent thereof. The values may be used to determine whether the section is cracked or uncracked at the nodes and then the effective value of D calculated. By introducing these values into the equations a new set of moments can be calculated and the process repeated until the moment variations are relatively small. In most practical cases, these will be no need to proceed beyond the first set of calculated moments.

DISADVANTAGES

1. From the values of Mx, My determined at each point, steel reinforcement can be provided for the full width of the grid (half the width on either side). Less steel will be required but the extra labour for detailing and fixing will only be justified in the case of large numbers of similar large panels with heavy loading.

2. Column stiffness cannot be taken into account directly unless the columns are so stiff that no rotation is possible along the column centre lines. Such cases are exceptions. In all other cases, the column Moments and rotation $\boxed{\theta_c} = M/4EKc$ must first be calculated from analysis by the Elastic Method. E.1, so that the deflections of the slab middle surface directly over the corners of the columns (e.g. Points 1 - 4) are known and can be used in the difference equations at adjacent nodal points in a finer mesh which picks up points in line with the column faces (e.g. Points 5, 6, 7 and 8).

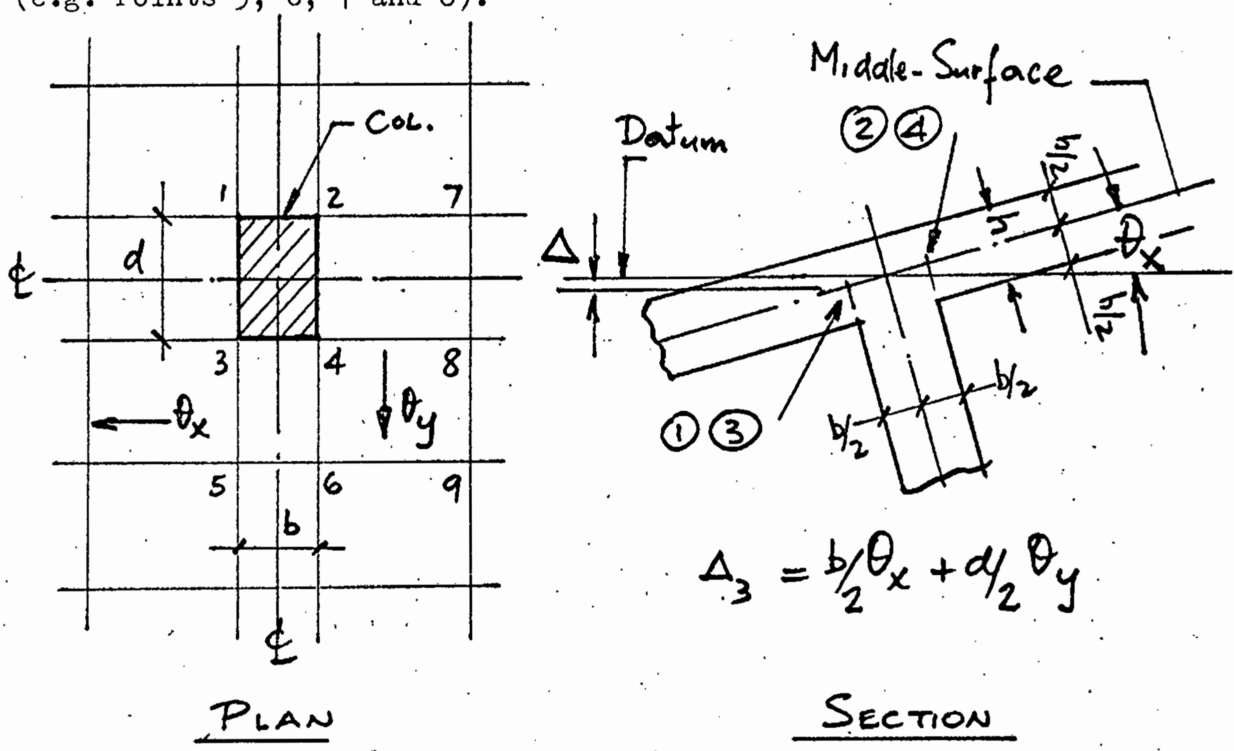


Fig. 44

This finer net also enables the determination of $(Mx)_5$ $(Mx)_6$ etc. which are different due to the column moment.

In general however, the method is best suited to simple column joints or slender columns.

APPLICATION

The use of this method will be illustrated in the analysis of a scale model in the following section.

H. EXPERIMENTAL FLAT PLATE

1. DESCRIPTION

An experimental slab representing a typical flat plate prototype to about one-third full size, was designed by the Elastic-Plastic Method in Section F 2.1, construdted and tested to destruction. The model consists of 4 rectangular bays each measuring 2,085 x 1,440 x 63 mm (aspect ratio $\frac{L}{5}$ = 1.45) freely supported on 9 columns.

2. DESIGN LOADING

A uniformly distributed load of 2.5 KN/m^2, (50 p.s.f.) together with line loading from 115 mm x 900 mm high brick walls along the column centre lines in the long direction and from one wall on the slab edge in the short direction. Details of the slab, supports, design loading and reinforcement layout are shown in Figures (45), (46) and (47)

3. SPECIFICATION

(A) CONCRETE

Strength : 30 MPa average at 7 days
Aggregate : 12 mm and 6 mm in the proportions of 30 : 70
Slump : 50 - 75 mm

(B) STEEL REINFORCEMENT

10 mm and 8 mm Mild Steel Rods with a minimum yield stress of 300 MPa (44,000 p.s.i.)

4. ERECTION PROCEDURE

The slab was cast on 20 mm "plydeck" formboard and approximately 150 x 150 x 10 mm thick rubber pads on mortar beds over the 9 columns. 8 mm cover to the reinforcement was provided by placing short lengths of this size of reinforcement under the bottom layer and 40 mm wood blocks under the first layer of the "top" reinforcement. The position of the slab was chosen so as to be centrally below two suspended hydraulic jacks each of 100 KN capacity, which could then be used to apply the loading to the top of the slab.

5. CONCRETE SAMPLE TESTS

The following samples were made for testing at the same time as the loading test :-

(a) 4 cubes 150x150x150 for crushing
(b) 2 cylinders 150 diameter x 300 long for tensile splitting

(c) /

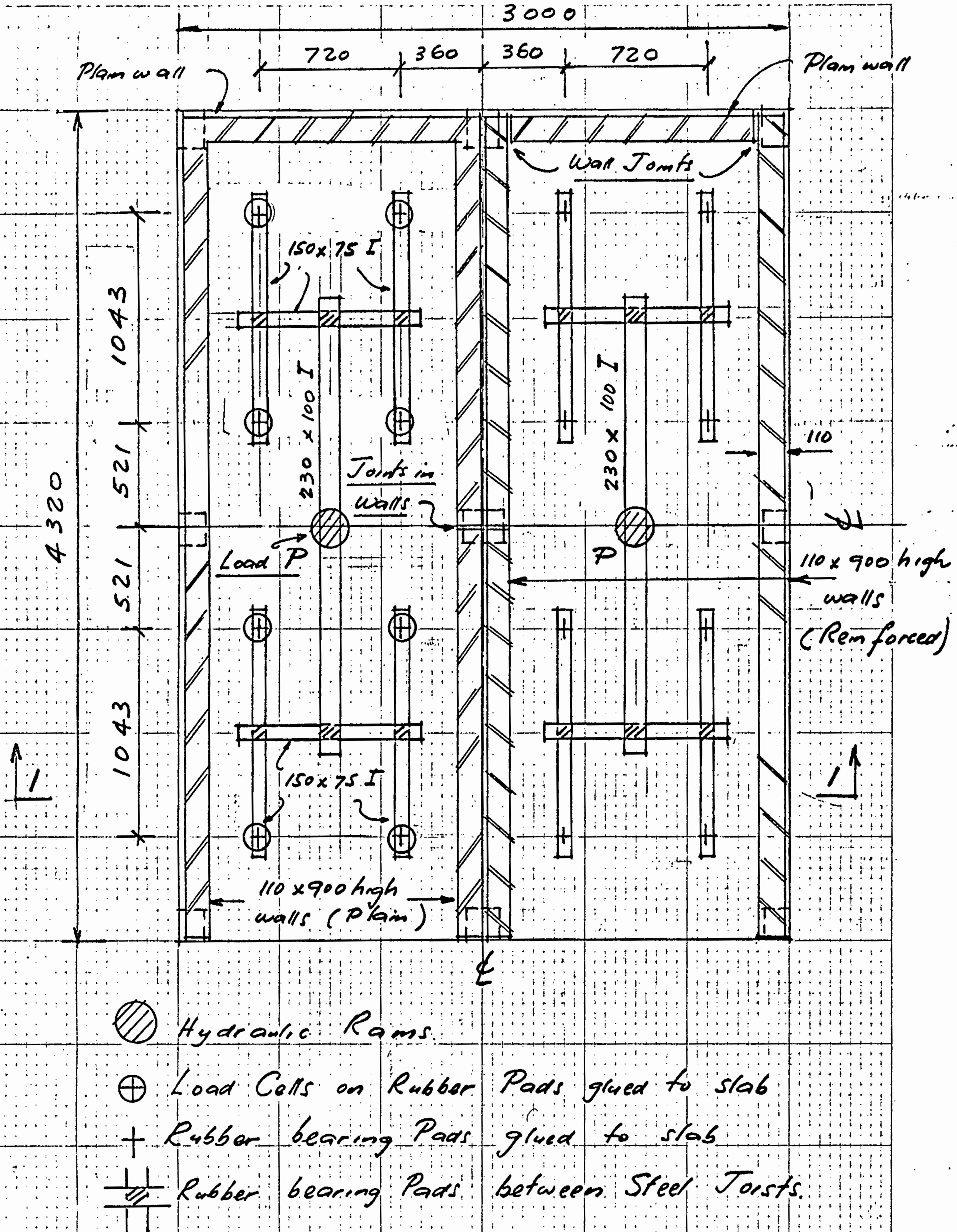


FIG. (45). PLAN OF SLAB SHEWING LOADING RIG AND BRICK WALLS.

NOTE: P = 6 KN produces average loading of 1KN/m²

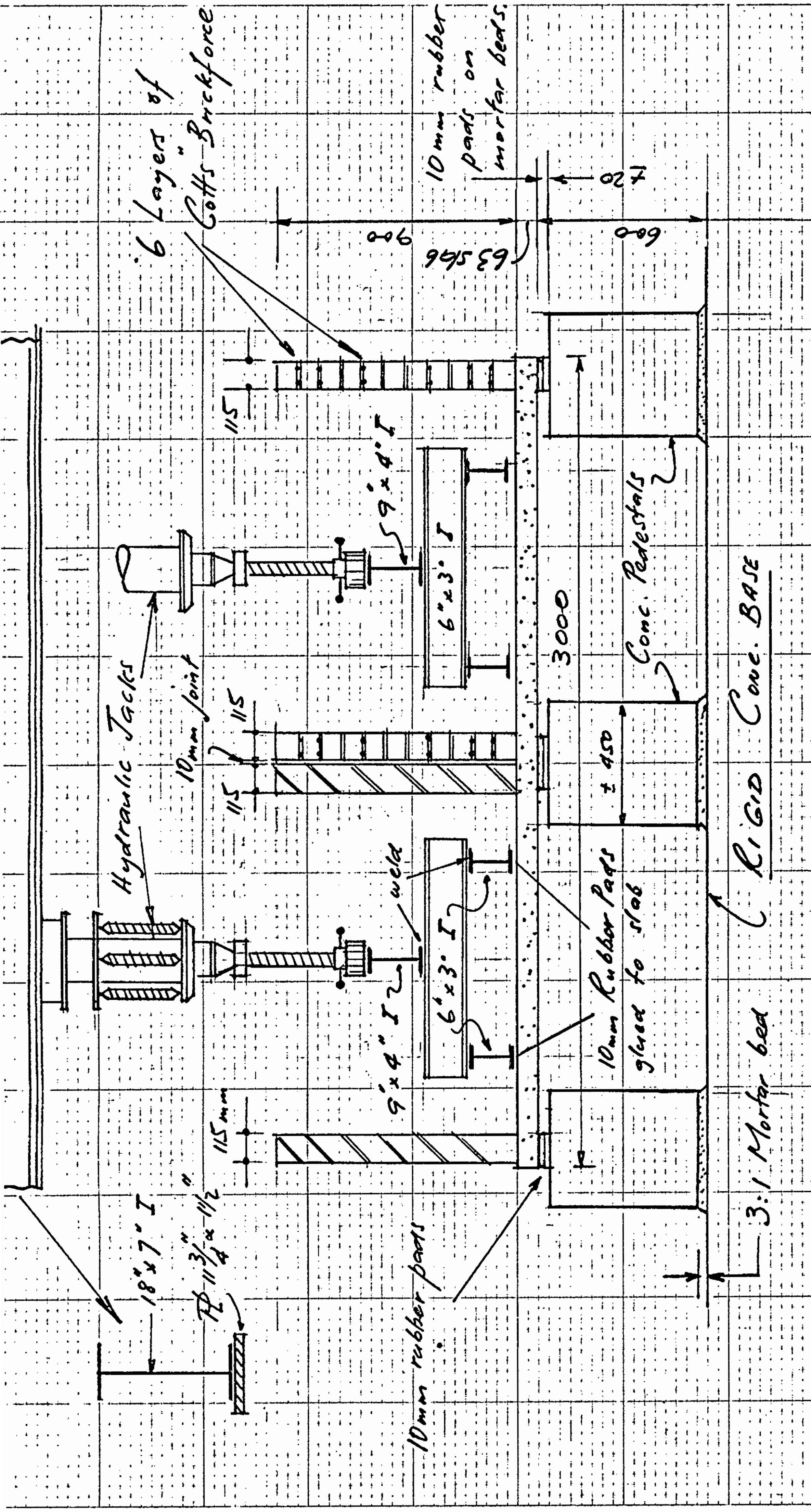


FIG 46 SECTIONAL ELEVATION 1-1 THRU' TEST SLAB

(See Fig 45)

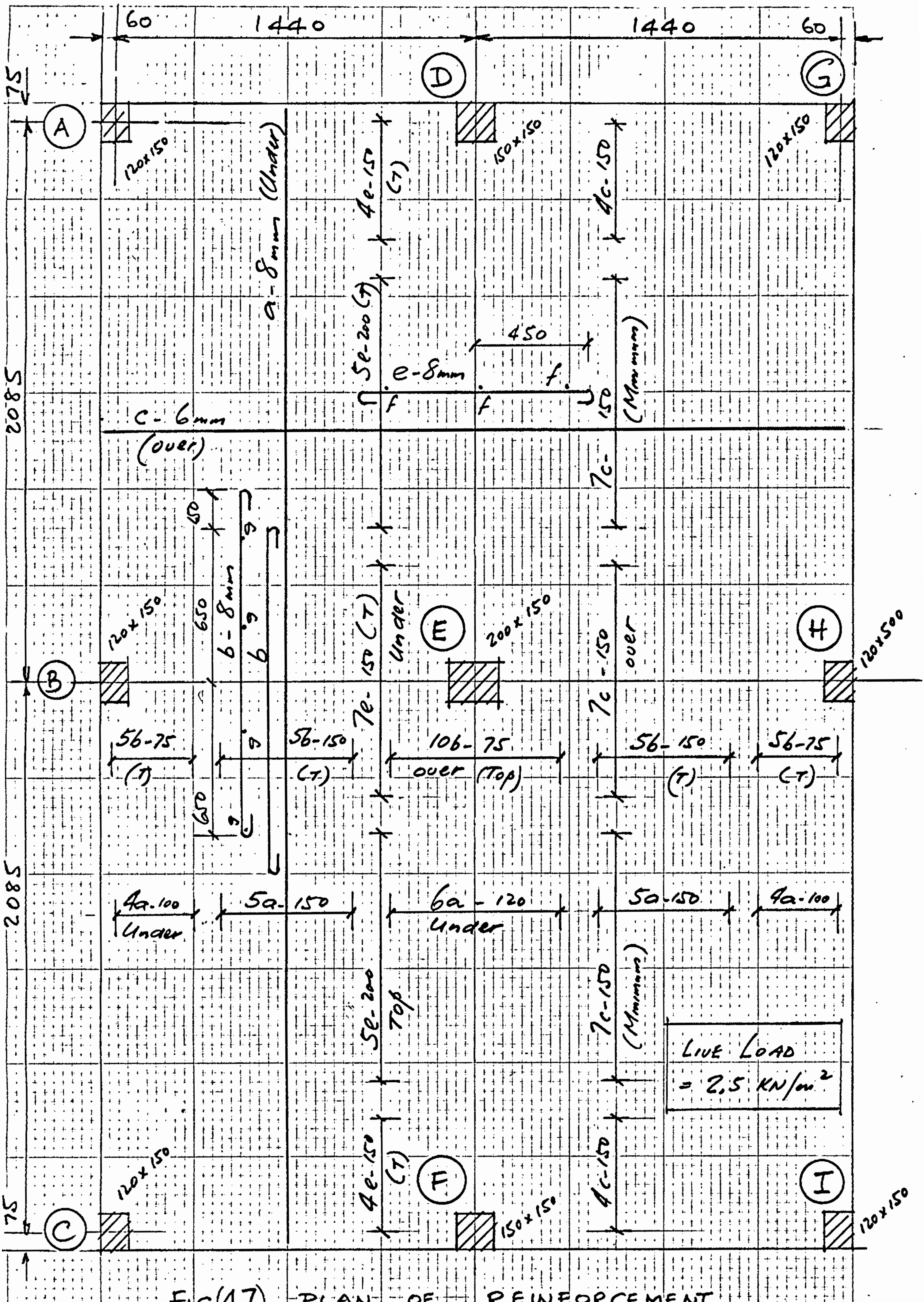


FIG.(47) PLAN OF REINFORCEMENT

(c) 2 beams 400x100x100 for measuring the Dynamic Modulus followed by bending to determine the modulus of rupture.

All samples were demoulded after 3 days and stored in a water tank under standard conditions till they were tested.

6. CURING

The slab was covered with black polythene sheeting for 3 days after casting, when it was removed to permit the building of 115 walls on the slab as shown. The walls were completed 7 days after the slab had been cast. The formwork and props were struck 21 days after casting.

7. LOADING ARRANGEMENT

The load from each jack was applied as a central point load to a 230x100 RSJ spanning between the centres of 2 adjacent panels and distributed from each end thereof through a grillage of short 150x75 R.S.J.'s to the one-quarter points of each panel, to simulate the effect of a distributed load on the panel bending moments and deflections as nearly as possible. As each jack could be individually controlled, it was possible to load each half of the total slab on either side of the long axis separately. However it was decided to always load both halves simultaneously to simulate a uniformly distributed load over the entire slab. 10 mm rubber pads (75x75) were placed at all pressure points between the steel girders and between the girders and the slab and glued to one side to allow for the relative rotations. To check the distribution of the load from each jack, "load-cells" were inserted between the rubber pads on the slab and the girder ends only in one half of the slab.

As stated previously 115 x 900 high brick walls had been built on the slab as shown in Figure (46) using stock R.O.K.'s and 4 : 1 cement mortar (No lime) and the effect of the weights of these walls would be additional to the effects of the loading from the jacks. Half of the walls were reinforced with "brick-force" galvanised reinforcement and half of the walls had 12 mm vertical expansion joints.

It was expected that at the beginning of the loading test, the walls would "arch" between supports and that the vertical loading intensity on the slab would be concentrated over or near the supports. Furthermore, this distribution of loading would probably alter with increasing slab deflection <u>after</u> the wall had cracked. For design purposes however, it was conservatively decided to assume that the weight of the wall was uniformly spread over the entire panel width.

8. INSTRUMENTATION

Dial guages were clamped to 50x50x6 mm steel angles independently spanning the full length of the slab and about 100 mm above it. These angles were stiffened by clamping them to inclined hangers suspended from the heavy steel cross-head supporting the jacks. Slab deflections were measured at the centres of all panels, midway between all supports and over the centre support to make allowance for the compression in the rubber pad.

Discs were glued to the outside of the external walls to measure horizontal strains on the centre-lines of the walls at the top of slab, top of wall and midway between the middle surface and the outer edges. Horizontal and vertical strains were also measured on the intersection of the face of the supports and the middle surface. The guage length was approximately 200. Dial guages were placed on the tops of the walls over the slab corners to measure slab uplift and against the outside face of the short wall to measure the wall rotations corresponding to the rotation of the edge of the slab.

9. PURPOSE OF TEST

The purpose was four-fold:-

(a) To verify the elastic behaviour of the slab as designed by the Elastic-Plastic method for loads approaching the Ultimate Load.

(b) To verify the Ultimate Load in Bending predicted from Yield-Line Theory.

(c) To compare the measured deflections with those predicted using the "Finite Difference" approximation to the Elastic theory equations for uncracked sections and therefore to check the accuracy of this method to predict both the deflections and the moments throughout.

(d) To observe the behaviour of the brick walls under increasing slab deflections including the strains at which cracking if any occurred in both the reinforced and unreinforced walls and to determine criteria if possible for indirectly preventing cracking.

10. "EXACT" ANALYSIS BY FINITE DIFFERENCES

Each panel was sub-divided as shown in Figure (48). into a 4 x 6 grid giving a mesh size of $\frac{1440}{4} \times \frac{2085}{6}$

i.e. 360x348. The mesh size was taken to be 354 mm square as

the actual /

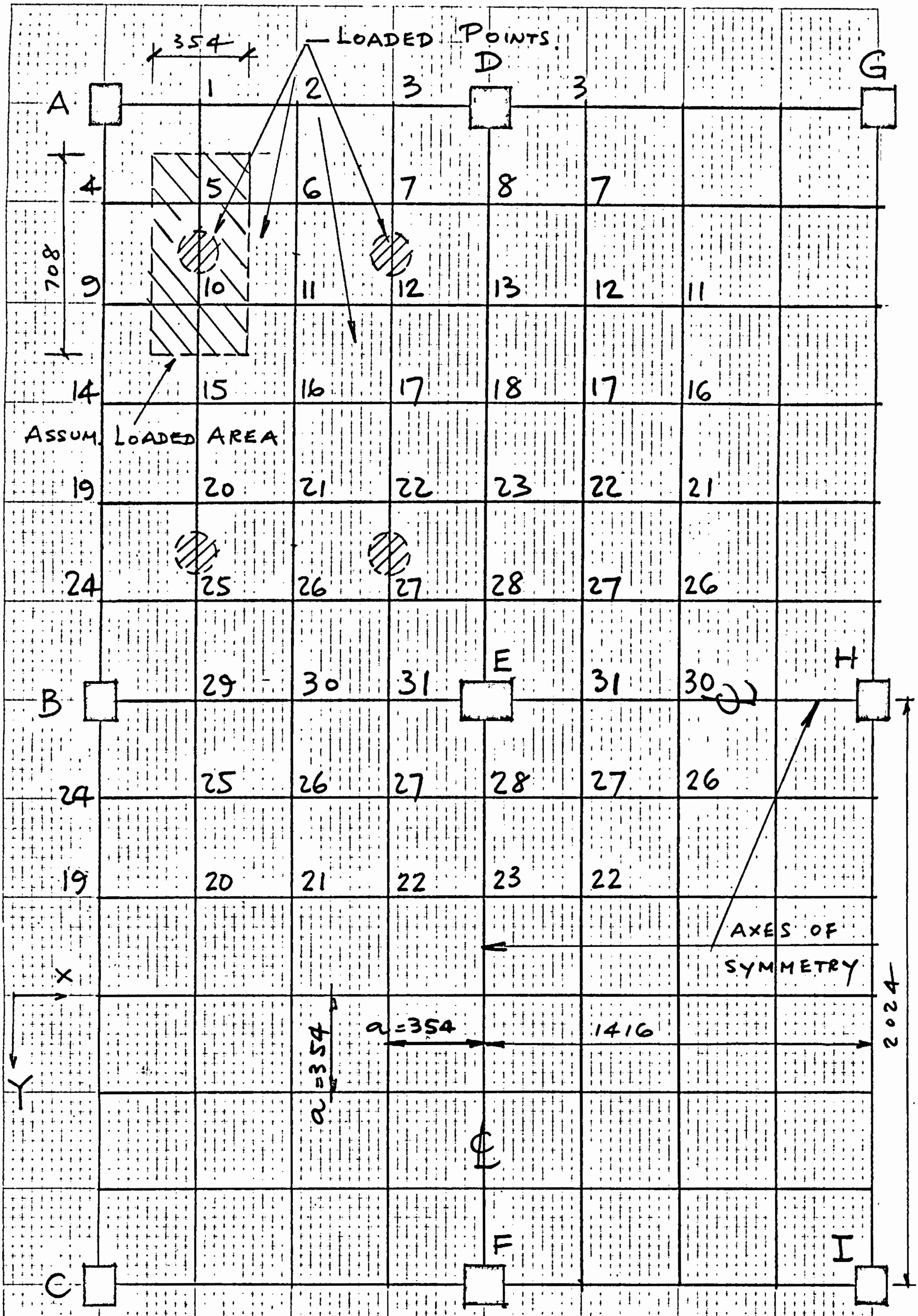
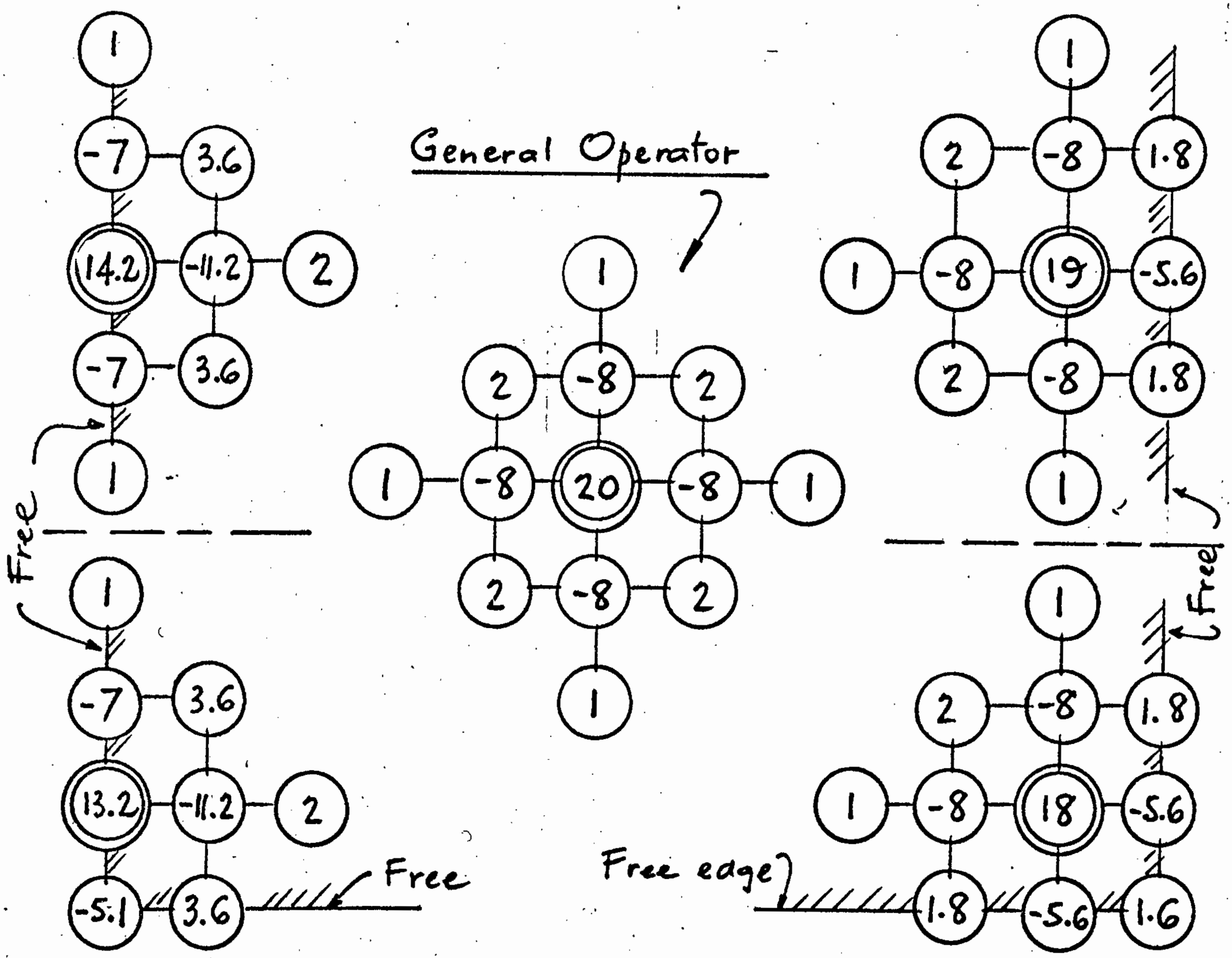


FIG 48 — FINITE DIFFERENCE MESH

actual lengths varied only $\frac{6}{360}$ i.e. 2% from the chosen size and therefore was unlikely to affect the accuracy. Poissons Ratio ν was assumed equal to 0.2.

As the loading would be symmetrical, only 31 nodal points were required. The various operator patterns then reduce to the following :-



The Matrix equations for the deflections w1 - w31 for the following loading conditions, are shown on pages 99-100.

(a) Unit Loading at nodal points
5, 7, 10, 12, 20, 22, 25, 27
to represent the equal applied loads at the quarter points.

(b) Unit Loading at all nodal points 1 - 31 to represent the uniform dead loading and for comparison with (a).

19	20	21	22	23	24	25	26	27	28	29	30	31					
														w_1		0	
														w_2		0	
														w_3		0	
														w_4		0	
														w_5		1	
														w_6		0	
														w_7		1	
														w_8		0	
1														w_9		0	
	1													w_{10}		1	
		1												w_{11}		0	
			1											w_{12}		1	
				1										w_{13}		0	
-7	3.6				1									w_{14}		0	
1.8	-8	2				1							×	w_{15}	=	0	$\frac{qa^4}{D}$
	2	-8	2				1							w_{16}		0	
		2	-8	2				1						w_{17}		0	
			4	-8					1					w_{18}		0	
14.2	-11.2	2			-7	3.6								w_{19}		0	
-5.6	19	-8	1		1.8	-8	2			1				w_{20}		1	
1	-8	20	-8	1		2	-8	2			1			w_{21}		0	
	1	-8	21	-8			2	-8	2			1		w_{22}		1	
		2	-16	20				4	-8					w_{23}		0	
-7	3.6				15.2	-11.2	2			3.6				w_{24}		0	
1.8	-8	2			-5.6	20	-8	1		-8	2			w_{25}		1	
	2	-8	2		1	-8	21	-8	1	2	-8	2		w_{26}		0	
		2	-8	2		1	-8	22	-8		2	-8		w_{27}		1	
			4	-8			2	-16	21			4		w_{28}		0	
	2				3.6	-16	4			19	-8	1		w_{29}		0	
		2				4	-16	4		-8	20	-8		w_{30}		0	
			2				4	-16	4	1	-8	21		w_{31}		0	

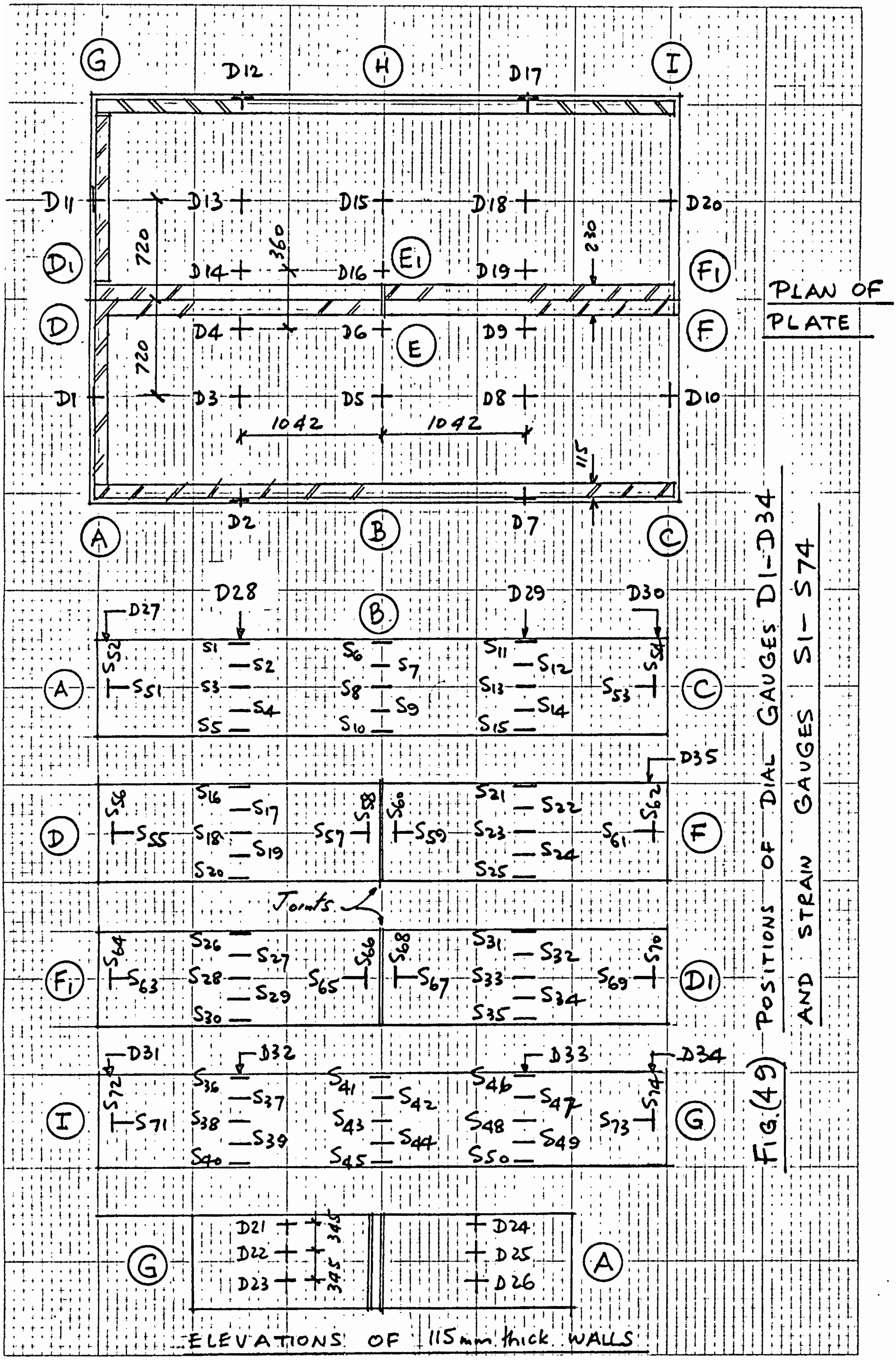


FIG. (49) POSITIONS OF DIAL GAUGES D1-D34 AND STRAIN GAUGES S1-S74

The equations were solved by computer using both the gross section stiffness and the cracked section stiffness. The following Load versus deflection graphs were then plotted at the nodal points corresponding to the positions of dial guages D1-D5, D7 - D10, D11 - D15, D17 - D20

(i) The calculated deflections due to slab dead load and the superimposed loads - using the gross section stiffness.

(ii) Ditto - using the cracked section stiffness.

(iii) The measured deflections due to the brickwork, slab and applied loads.

11. EFFECT OF WEIGHT OF BRICKWORK

From the above graphs, it was hoped to make some assessment of the following :-

(i) Distribution of vertical load from the walls to the slab.

(ii) Additional Moments in the slab due to the walls.

(iii) Additional Deflections in the slab due to the walls.

12. PLATE FLEXURAL RIGIDITY (STIFFNESS)

$$Dg = \frac{Ech^3}{12(1-\nu^2)} \qquad \text{.....................(66)}$$

where Dg = Flexural Rigidity of the uncracked section

Ec = Instantaneous Elastic Modulus = $57,600\sqrt{Fc^1}$ p.s.i.

h = depth of slab

ν = Poissons Ratio — 0.2 (approx.)

$$Dcr = \frac{Espd^3(1-K)(1-\frac{K}{3})}{(1-\nu^2)} \qquad \text{........................(67)}$$

where Dcr = Flexural Rigidity of the cracked section

Es = Elastic Modulus of the Steel

n = Es/Ec

p = reinforcement ratio

d = effective depth of the reinforcement

K = depth of Neutral Axis

$= \sqrt{pn\ (2 + pn)} - pn$

For the Test Slab Dg = 6.7×10^8 ($N.mm^2$ per mm

and Dcr = 1.4×10^8 " (Long Direction)

13. TESTING PROCEDURE

The numbered positions of dial guages and wall strain guages are shown in Fig. (49).

Immediately before and after stripping of the slab, the dial guages were read and the wall strains were measured. Equal loads were then applied from each Jack up to a maximum of 60 KN per Jack (60% of full capacity) in steps of 10 KN applied at the slowest possible rate, all dials being read after each step. Each step lasted approximately 20 minutes. At the end of loading (referred to hereinafter as the first loading) the average load applied was 10 KN/m^2. This loading was then sustained for a period of 40 hours after which it was removed. Four hours later the dial and strain guages were again read and the second loading was commenced.

During the second loading, the loads from each Jack were increased in steps of 30 KN per Jack (5 KN/m^2) to 60 KN followed by 80 KN, 90 KN and then to 100 KN at which load flexural failure occurred in the two panels B C I H (Fig. 48). Where possible dial readings were taken at the end of each loading stage.

As wall strains were not expected to be large, readings thereof were only taken, where possible, during the second loading at the 30 KN and 60 KN stages.

I. RESULTS OF TESTS

1. TESTS ON CONCRETE SPECIMENS (See Section 5)

The following average results were obtained after 28 days (day of slab test).

(a) Cube crushing (6 samples) - 5150 p.s.i. (35,4 MPa)

(b) Cylinder splitting (1 sample) - 420 p.s.i. (2,9 MPa)

(c) Dynamic Modulus (3 samples) - $5{,}4 \times 10^6$ p.s.i.

From the above the Elastic Modulus $E_c = 4{,}4 \times 10^6$ p.s.i. (30 000 MPa)

2. TESTS ON 8mm DIAMETER STEEL ROD SPECIMENS

Four specimens were tested and had an average breaking strength of 65,000 p.s.i. at a strain of about 20%. Yield appeared to start at ± 52,000 p.s.i. (360 MPa) and this stress was used in the Ultimate Moment Calculations although it may have been conservative.

3. PREPARATION OF LOAD-DEFLECTION CURVES

Due to the large amount of work entailed curves were plotted only for the following Dial guages.

Dial	Corresponding Nodes N	Figure
D9, D19	Average of N17 N18	50
D8, D18	N16	51
D5, D15	N30	52
D3, D13	N16	53
D4, D14	Average of N17 N18	54
D6, D16	Support E	55
D10, D20	N2	56
D17	N14	57
D12	N14	58
D21, D22, D23	Rotation wall D-G	59

As the dials placed together in the left hand column represented similar Nodes, straight lines with the closest fit were drawn through all the points plotted for the respective loadings. In general, both sets of points were very close together in the first loading test, but deviated slightly in the second loading test. It was preferred to draw the graphs as a series of best fitting straight lines rather than curves.

Only /

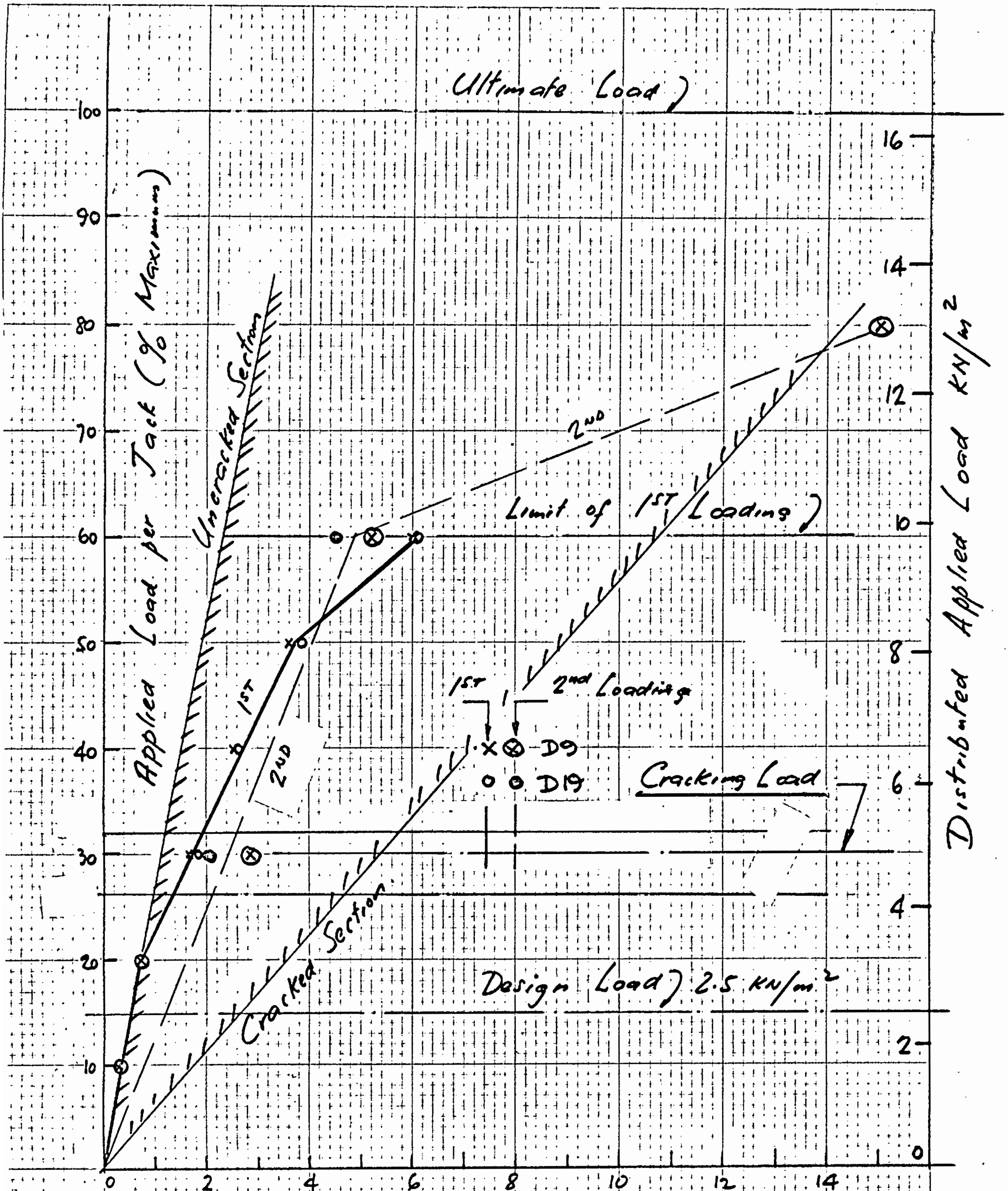


FIG. 50 – Incremental Deflection (mm)

– D9 and D19 –

NOTE: 1) Load per Jack = 2 x Panel Load

2) Corrected deflections are less than shewn.

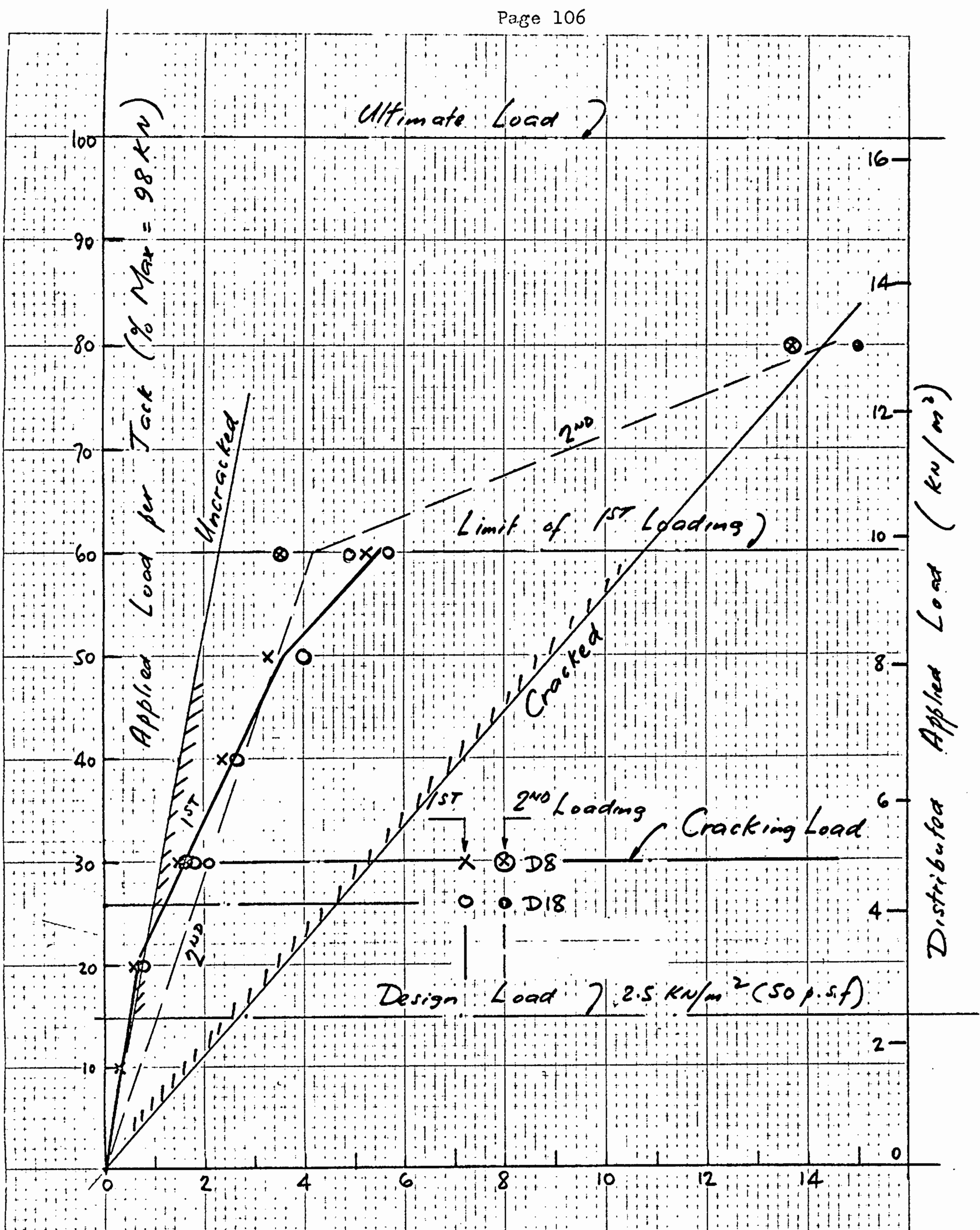


FIG. 51 – Incremental Deflection (mm)

– D8 and D18 –

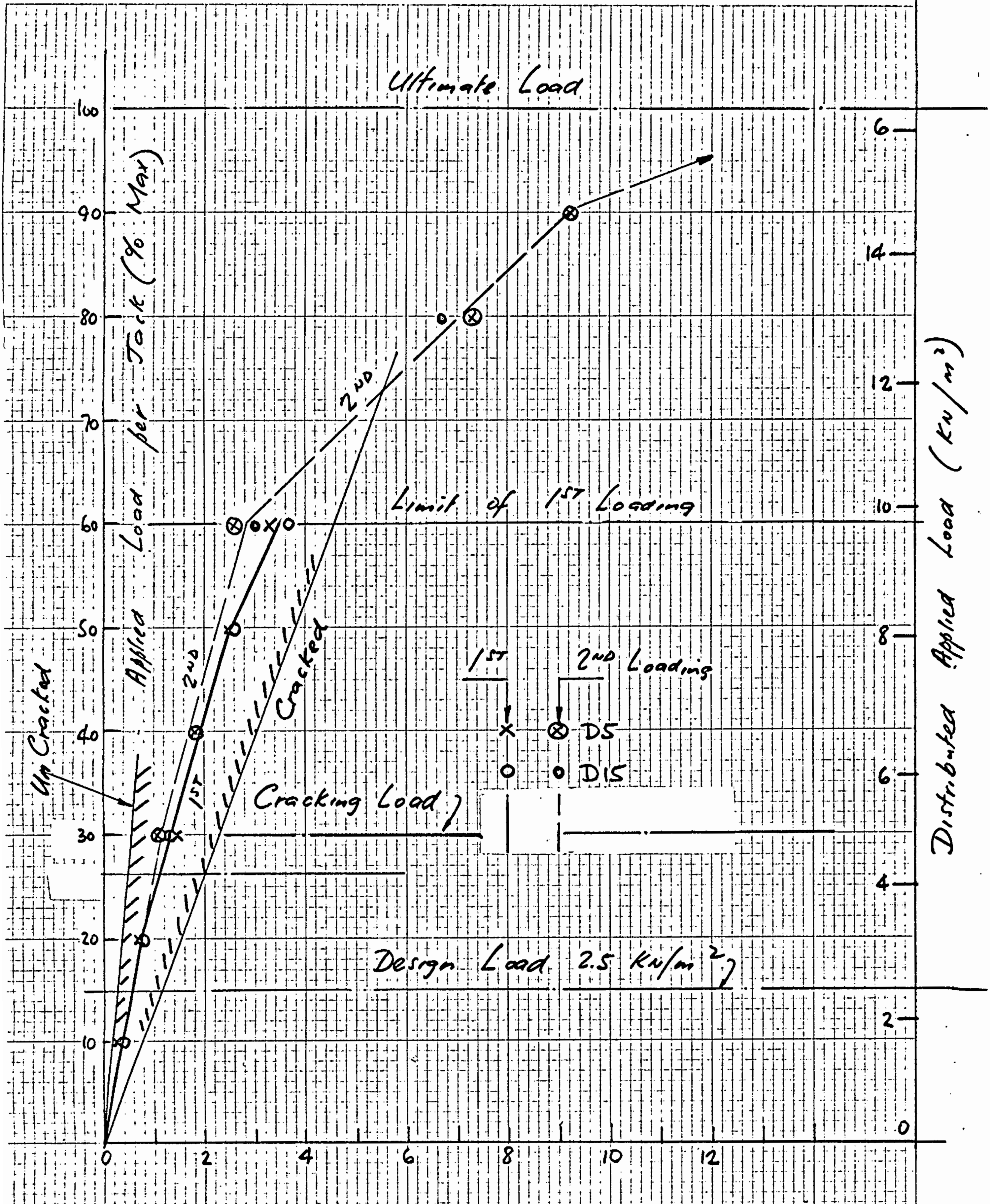


FIG. 52 - Incremental Deflection
- D5 and D15 -

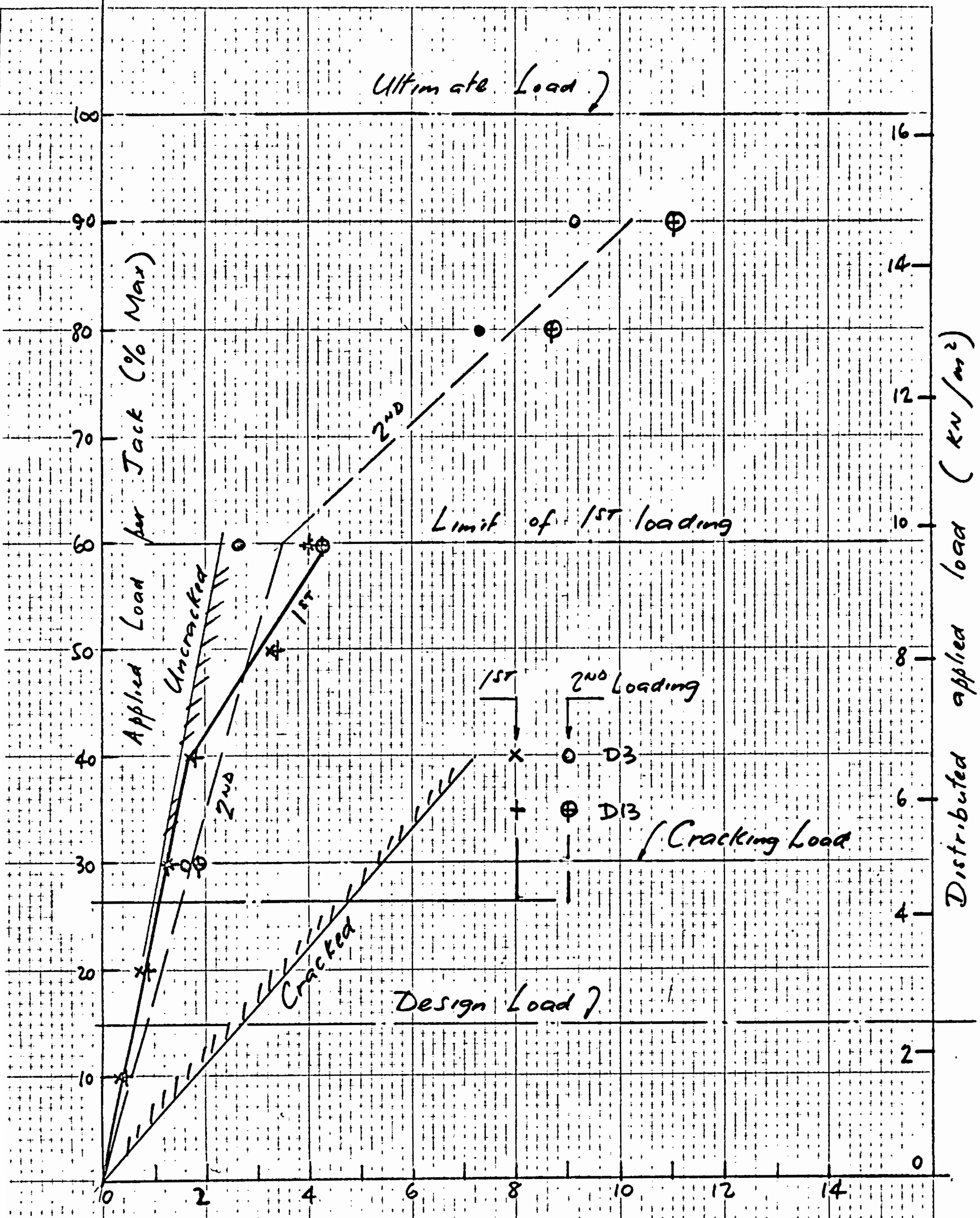


FIG 53 - Incremental Deflection (mm)

- D3 and D13 -

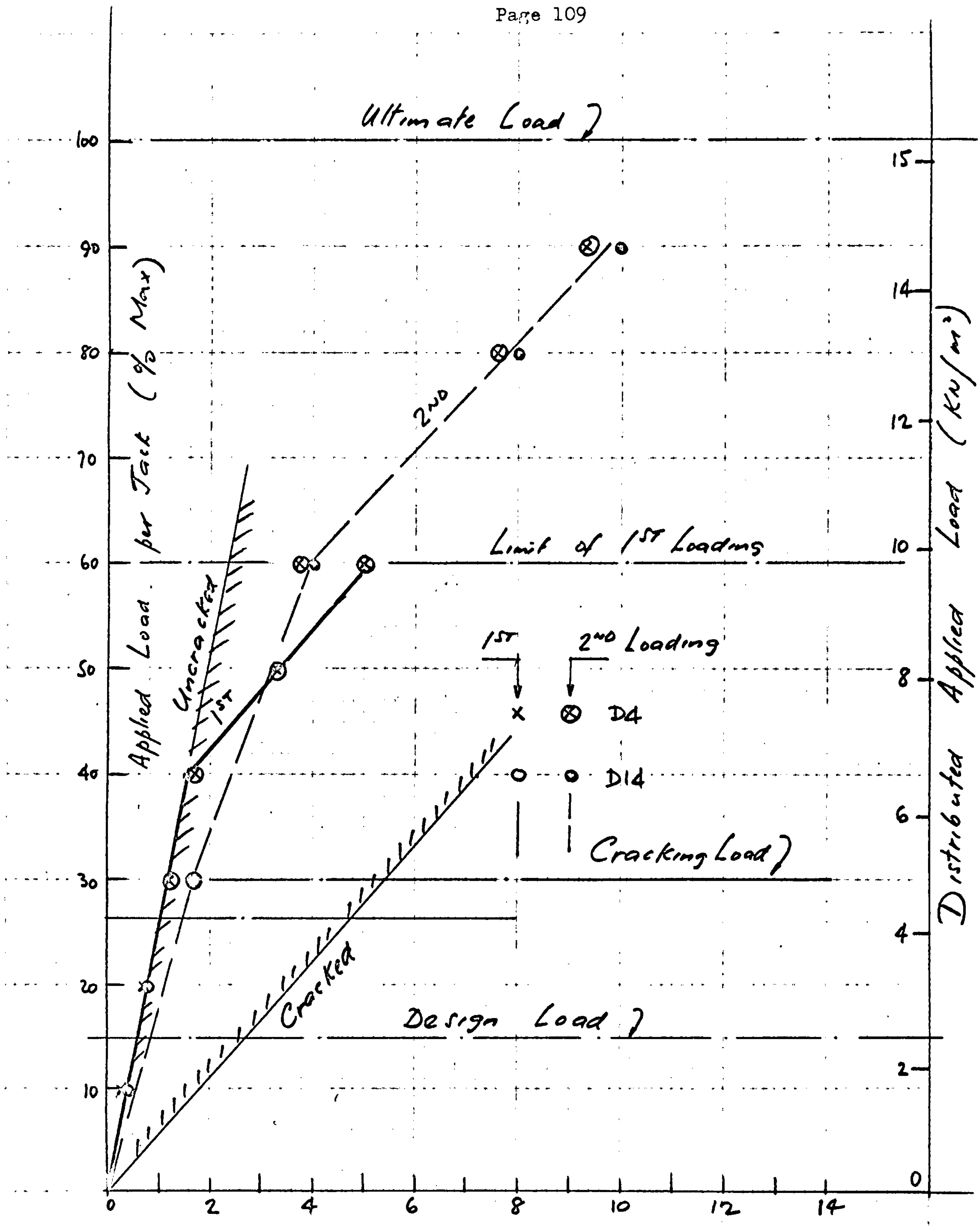


FIG 54 - Incremental Deflection (mm)

- D4 and D14 -

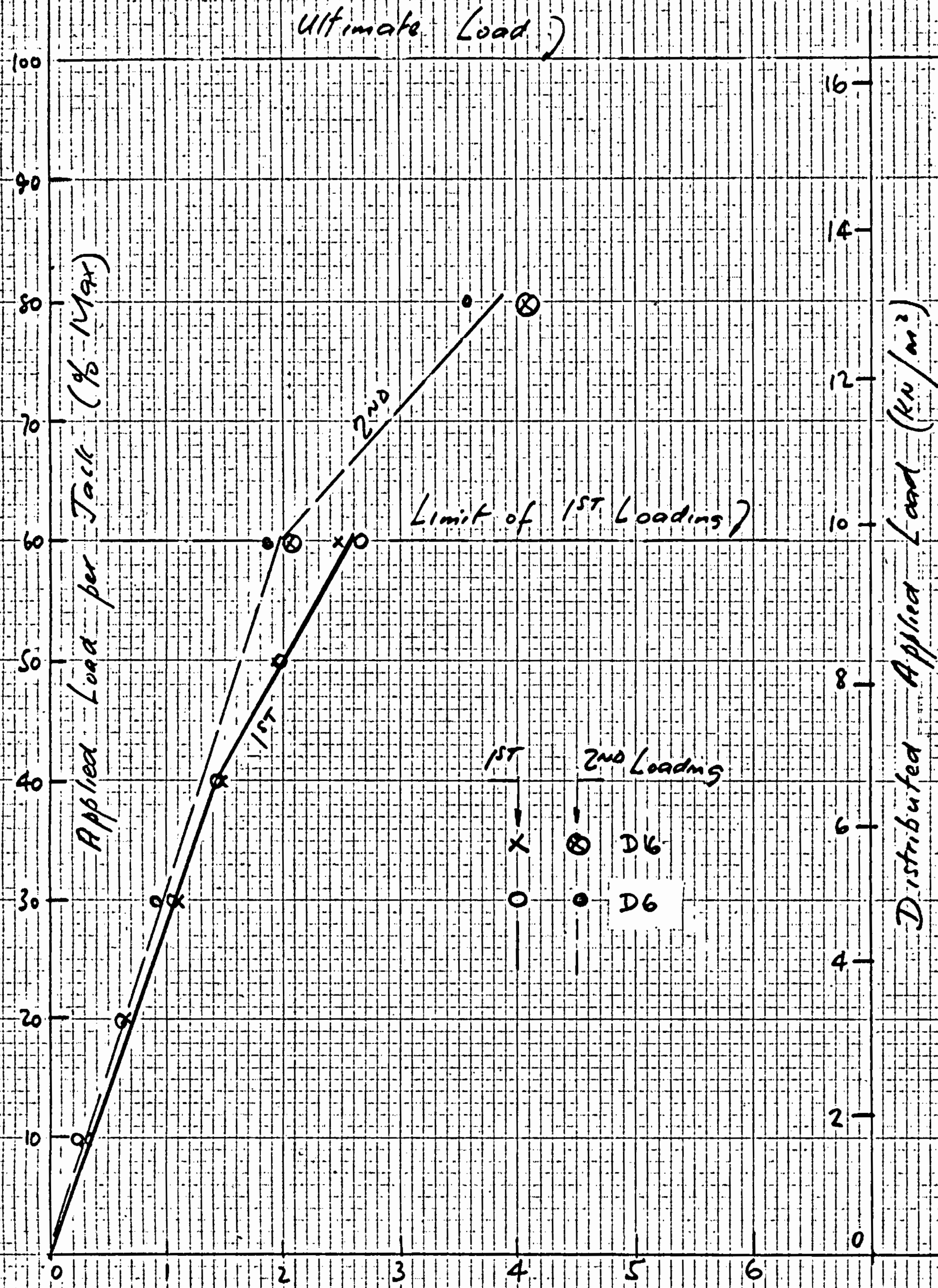


FIG 55 - Incremental Deflection near Centre Support

- D6 and D16 -

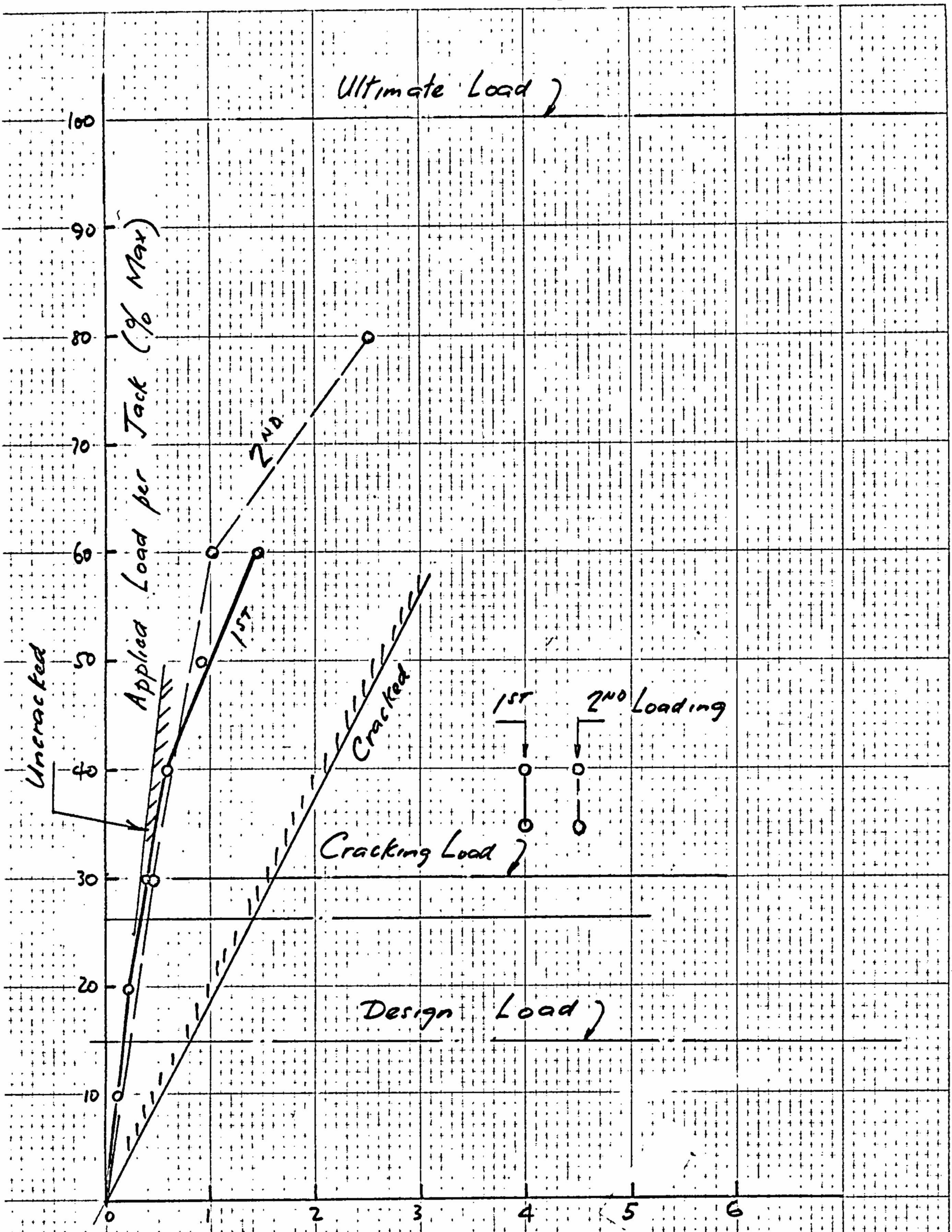


FIG 56 - Incremental Deflection (mm)

- D10 and D20 - (Average)

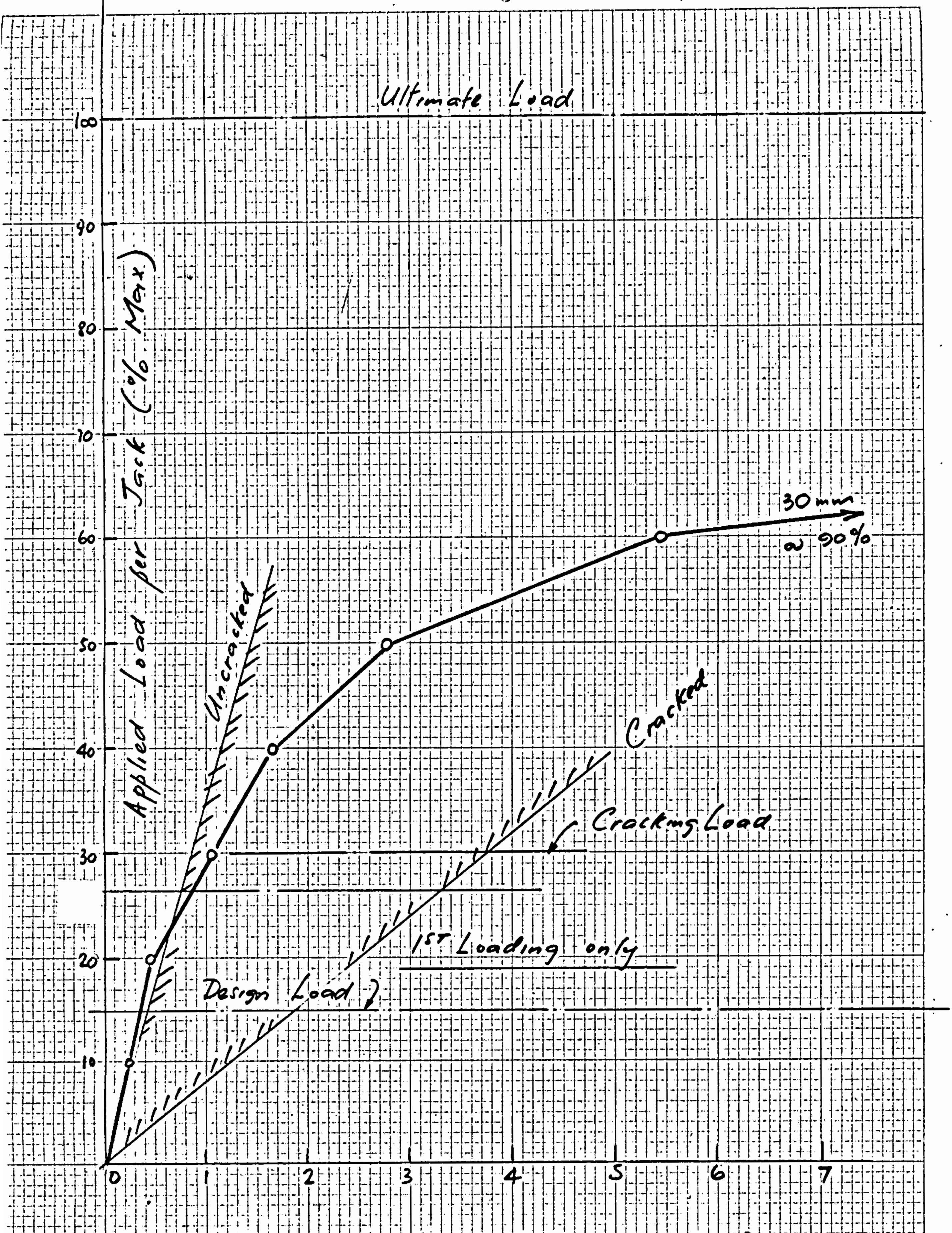


FIG 57- Incremental Deflection (mm)

- D17 -

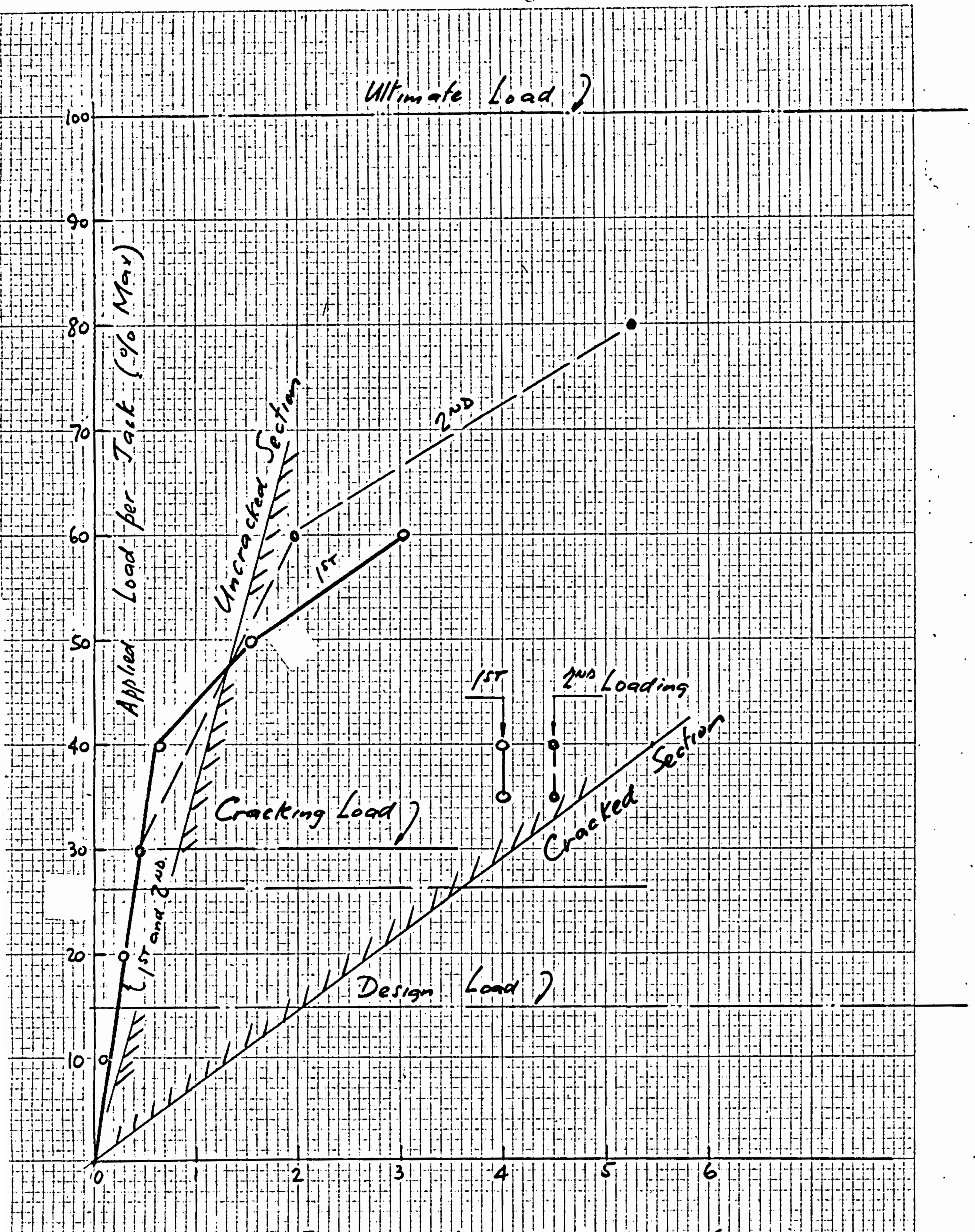


FIG 58- Incremental Deflection (mm)

- D12 -

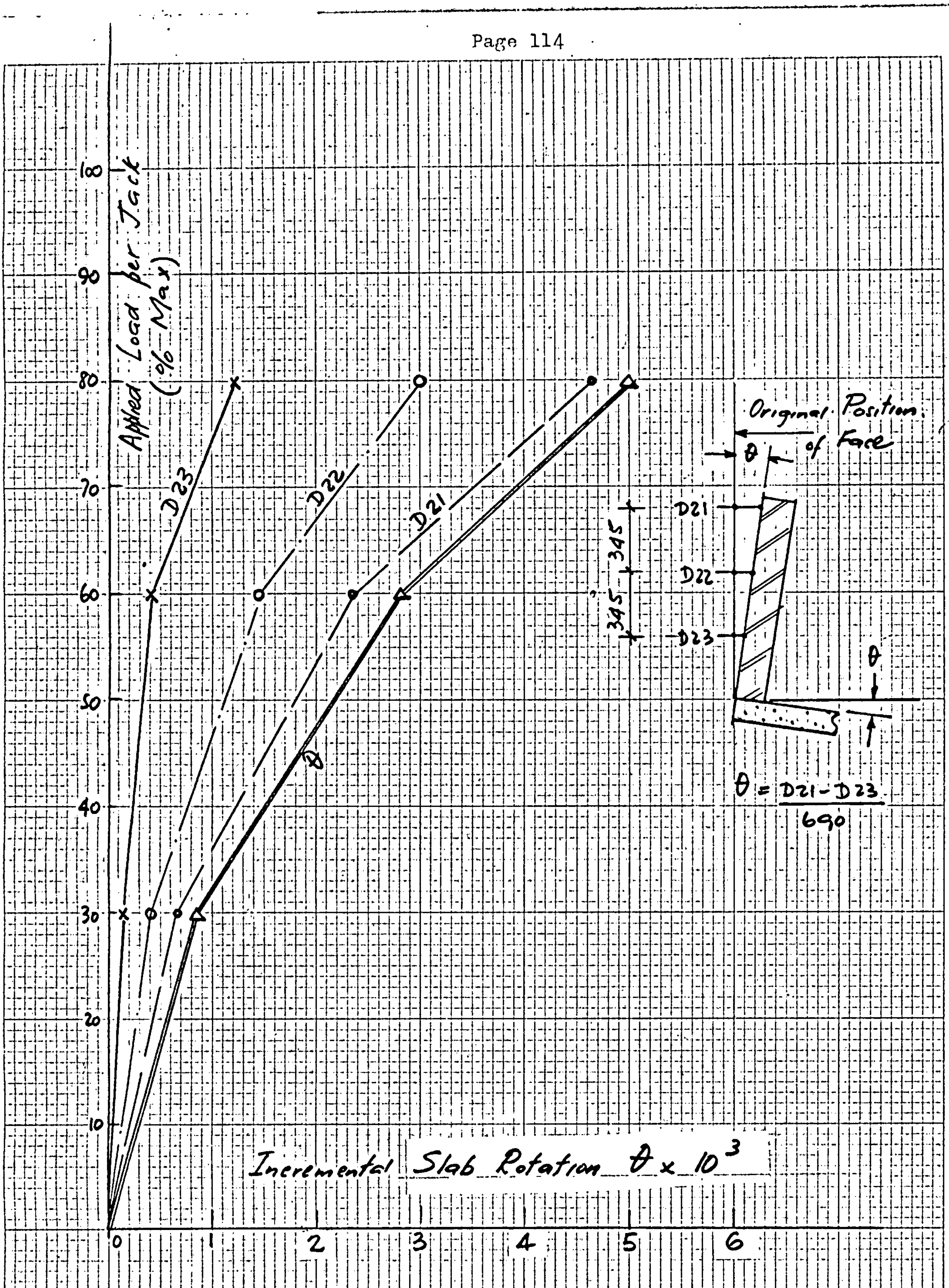


FIG 59 - Incremental Lateral Deflection (mm)

- D21, D22, D23 -

Only incremental deflections due to the applied loads from the hydraulic jacks were plotted, no account being taken of the effects of the slab dead load, walls and loading girders under the jacks.

From the computer print out of deflection in terms of $\frac{qa^4}{D}$ the load versus deflection straight line graphs were drawn for both D equal to the gross section stiffness Dg and the cracked section stiffness Dcr respectively. In calculating Dcr, the average percentage of bottom reinforcement in the entire panel width in the long direction was used viz. $p = 0.008$. The ratio $\frac{Dg}{Dcr}$ is nearly equal to 5.

4. CORRECTIONS TO MEASURED DEFLECTIONS

(a). SETTLEMENT OF SUPPORT E

As shown from D6 and D16 (Figure 55) Support E settled due to compression of the rubber pad. The settlement was linear up to 60% of the failure load when it reached 2 mm. As the settlements of the other supports were small by comparison, the true deflection at any node, say N16, for this load, would be less than that measured, by the deflection of N16 when the node E is displaced 2mm by a concentrated load applied at E to the panel supported as before but not at E. (See Figure 60)

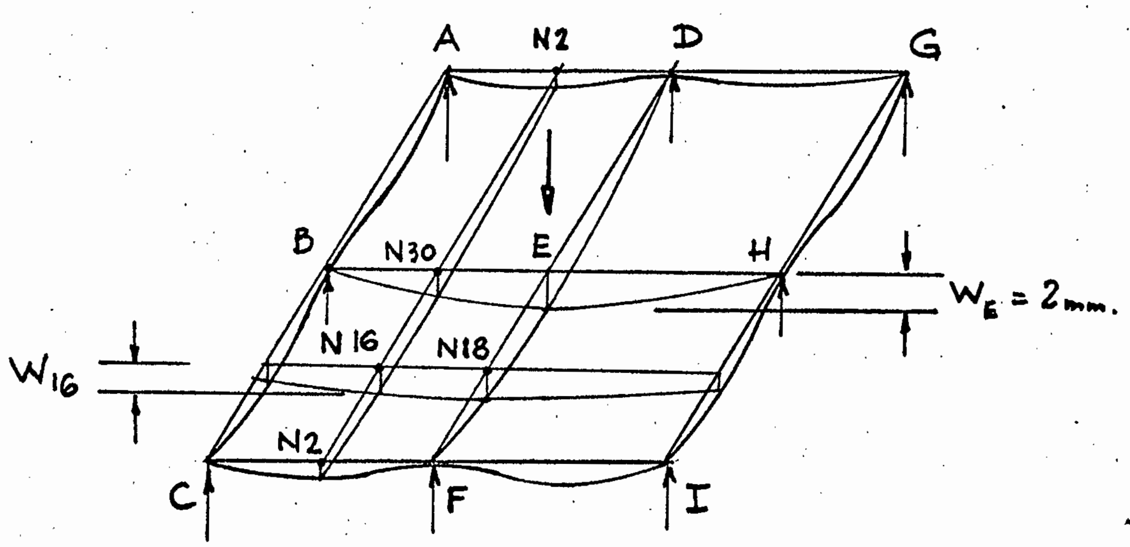


Fig. 60

Therefore apply /

Therefore apply unit load at E, calculate the relative deflections everywhere by solving 32 Finite Difference Equations and apply corrections at all the nodes in proportion to the known value of W_E

(b) The steel girder supporting the hydraulic jacks as well as the steel framework to which all the centre dials were clamped viz. D3-D6, D8-D9, D13-D19, is a 18" x 7" R.S.J. with a welded bottom plate $11\frac{3}{4}$" x $1\frac{1}{2}$" and spans 11' 0". (See Figure 61 below)

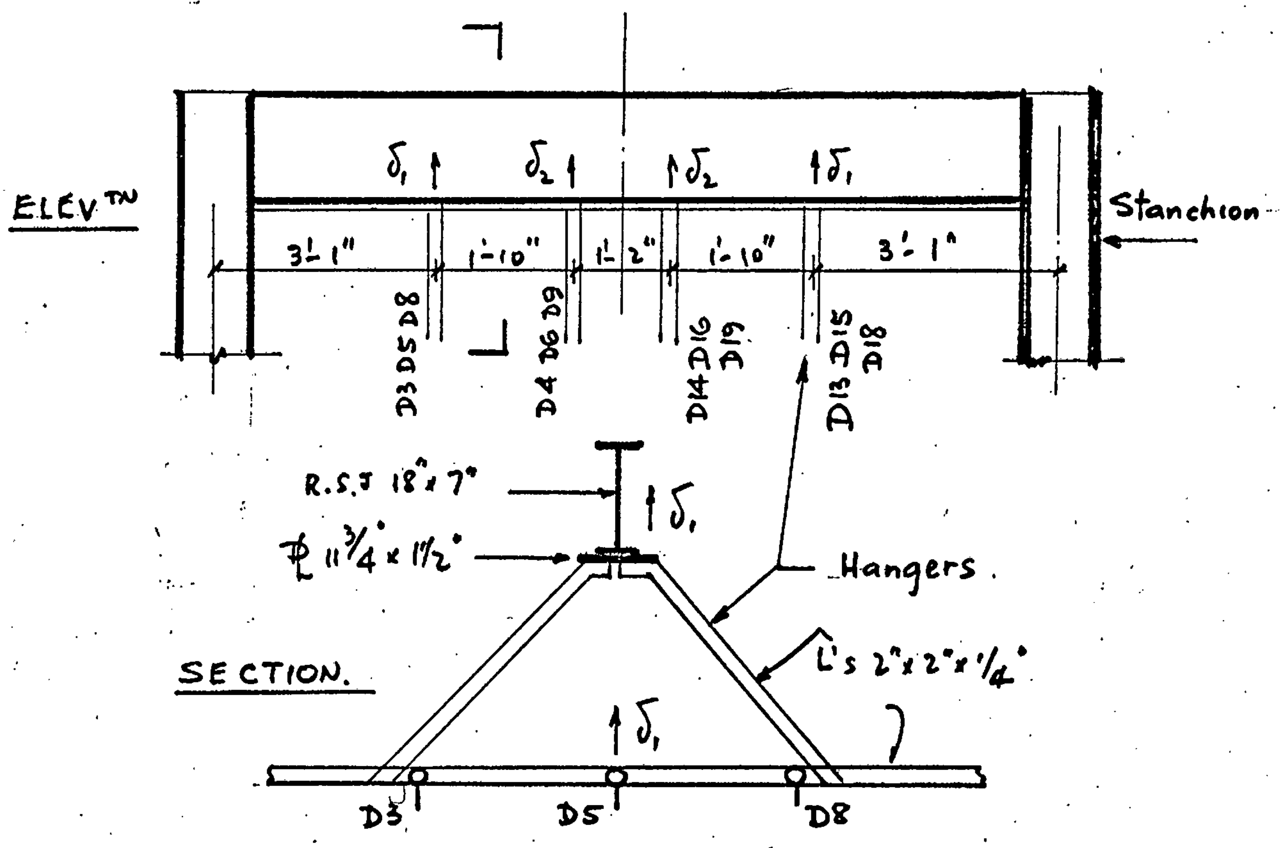


Fig. 61

Due to the jacking loads, an upward deflection was caused at each hangar position which can be calculated. Hence all the dial guages deflect upward by the same amount which should be deducted from the readings. This correction is also proportional to the jacking load.

The total correction to each dial reading is therefore proportional to the applied load and if deducted would only cause a rotation of the graph about the origin so as to reduce deflections. However, the graphs were not corrected as the corrections were small and as the shape of the graph was considered to be more significant than the correct position.

5. CRACKING LOAD

The elevation of the longer spans and the cross-section at L/4 at L/4 where the positive moment reaches a maximum, is shown in Figure 62.

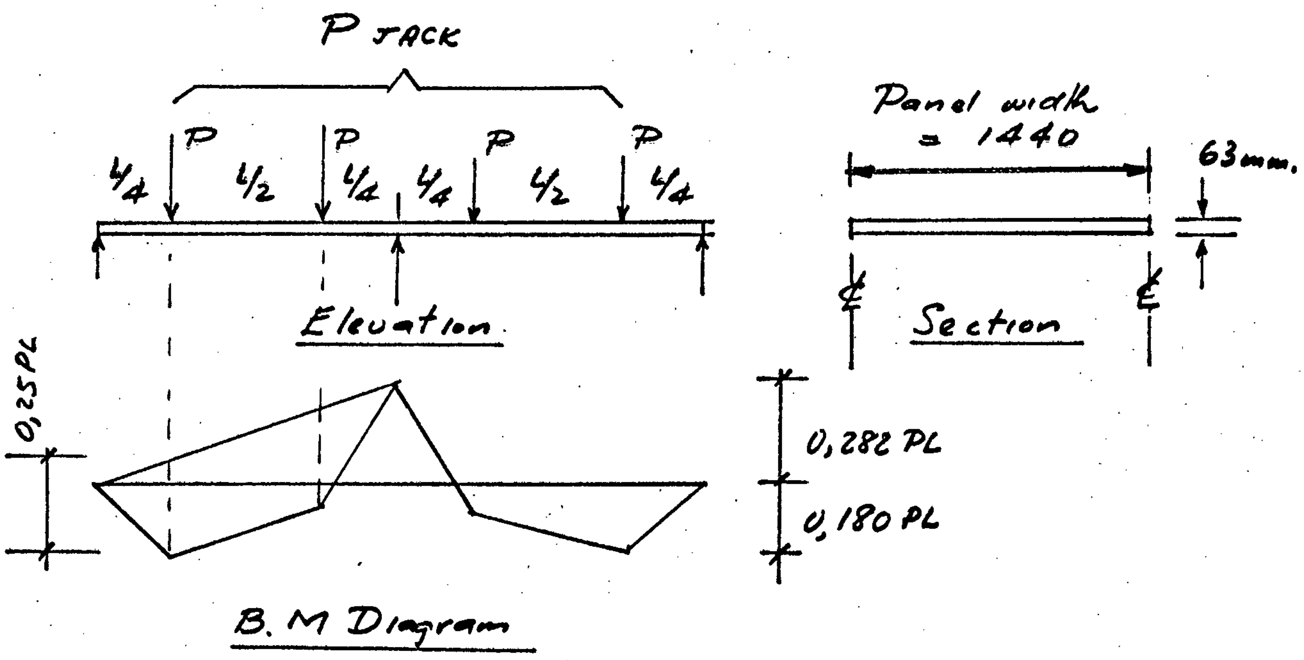


Fig. 62

If the Modulus of Rupture is taken as $8\sqrt{Fc^1}$ p.s.i.
= 510 p.s.i. = 3,5 MPa

$$Mcr = fZ = 3{,}5 \times \frac{1}{6} \times 1440 \times 63^2 = 3{,}34 \text{ Kn.m.}$$

Less Moment due to slab dead load at L/4	= 0,57
Net Moment	= 2,77 KN.m
Moment due to P	= 0,18 PL

Therefore P = 7,4 KN

Cracking Load P_J = 4P = 29,6 KN = 30% Failure Load

Similarly the cracking load at the support section is approximately 14% of the failure load. The cracking load for the bottom of the slab is shown on each graph to assess the slab behaviour for loads less or greater than the cracking load.

6. SLAB BEHAVIOUR

(a) FIRST LOADING

Out of the 14 central nodes investigated 50% deflected according to the slab's uncracked stiffness for loads up to 20% of the Failure load and 50% of nodes for loads up to 40% of the Failure load, or nearly $7KN/m^2$. The changes in the slope at the 20% and 40% levels are due to the spread of cracks throughout the slab particularly near mid-span. At the 60% level, P = 15 KN and the total slab negative moment is

$$M = 0{,}282\ PL + 0{,}125\ WL$$
$$= 0{,}282 \times 15 \times 2{,}085 + 0{,}125 \times 1{,}5 \times 1{,}440 \times 2{,}085^2$$
$$= 10\ KN.m$$

From the reinforcement provided

$$-\ Mult = 11{,}2\ KN.m$$

Similarly the total slab positive moment is

$$M = 0{,}18\ PL + 0{,}57 = 6{,}2\ KN.m$$
$$\text{and } +\ Mult = 8{,}9\ KN.m$$

Theoretically therefore, the steel of both Critical Sections had nearly reached its elastic limit. This was confirmed in the test as the slab had deflected elastically at all the nodes up to the 60% level (10 KN/m^2).

(b) SECOND LOADING

On removal of the load and re-loading to failure, the slab initially behaved elastically at all the nodes, with a constant stiffness at any node, so that it reached a slightly SMALLER deflection than before at the 60% level. In all cases except D17, the effective slab stiffness was greater than the cracked section stiffness for loads less than 75% of failure and in all cases for loads less than 60% which in fact represents 75% of the design ultimate load.

As expected, for loads less than the cracking load, the gross section stiffness can be used to predict short-term deflections very closely.

With the load at 80%, deflections had increased non-elastically due to yield of the negative reinforcement so that visible cracks occurred in the top surface over the columns and parallel to BEH, and wider cracks in the bottom surface on either side of and parallel to the loaded areas nearest to supports CFI. This concentration of cracking in the region of the section having the maximum positive moment, was accompanied by a concentration of curvature, so that the slab distinctly folded in this region from one edge to the other. In other words, a positive yield line had formed at this section.

A crack also formed in a wide arc around side column H.

At the 90% level, deflections increased further without noticeable increase in the crack widths. Some of the guages e.g. D8, D9, D18, D19 lost contact with the slab and could not be read further.

At the 100% level, failure commenced by spalling of

the/

the concrete in the top surface at the section of maximum positive moment where the slab had earlier found a yield line. A few minutes later a punching shear failure occurred around the rubber pad at B accompanied by a sharp crack.

Details of these and other cracks in both surfaces which occurred just before or at failure are shown in Figures 63 and 64. Note the conical or near conical crack patterns around columns A, C and H.

For some inexplicable reason, deflection and cracking were far more severe in panels B-C-I-H than in panels B-A-G-H. This may be due to either variations in the concrete quality or bond slip. Despite the heavy cracking over the internal supports, the slab curvature was not excessive and so a yield line had not yet formed there. Consequently it would have been possible for the slab to carry further load if shear failure had not occurred but this additional load would have been small as shown later in the calculation of the Ultimate Load.

7 YIELD OF REINFORCEMENT AT COLUMN EDGE

(R.4.) This was expected from theory to occur when the panel load

$$P = \frac{-4\pi M_{py}}{\log_e \frac{r_b}{L_x} + 2{,}31 - \frac{L_y}{L_x}} \qquad \ldots\ldots\ldots\ldots(55)$$

$M_{py} = p.f_y \times 0{,}9d^2$ where p = reinforcement ratio over the column.

$$M_{py} = \frac{50}{75 \times 48} \times \frac{360 \times 0{,}9 \times 48^2}{10^3} = 10{,}4 \text{ KN}$$

From (55) $P = \dfrac{-4\pi \times 10{,}4}{\log_e \frac{85}{2000} + 2{,}310 - 1{,}4} = 54$ KN

Deduct weight of slab and girders	= 5,5
Applied Load per panel	≑ 48 KN

Although strict speaking equation (55) only applies to a uniformly loaded internal panel, it was used as a rough guide to show that yielding at the column edge would occur at loads near to the design ultimate load. It was therefore anticipated and confirmed by the graphs

that the /

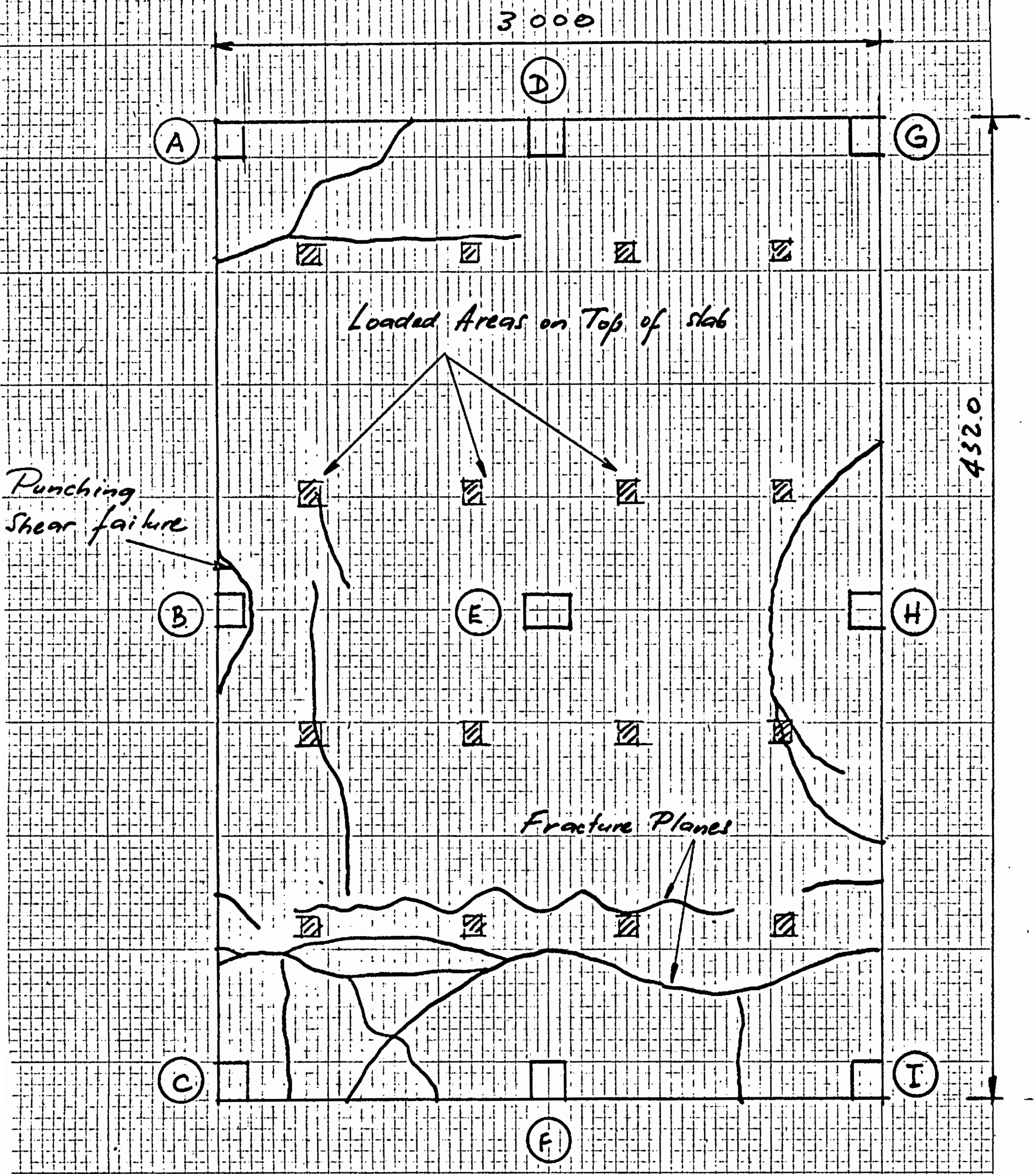


FIG 63 CRACK PATTERN IN BOTTOM OF SLAB AT FAILURE

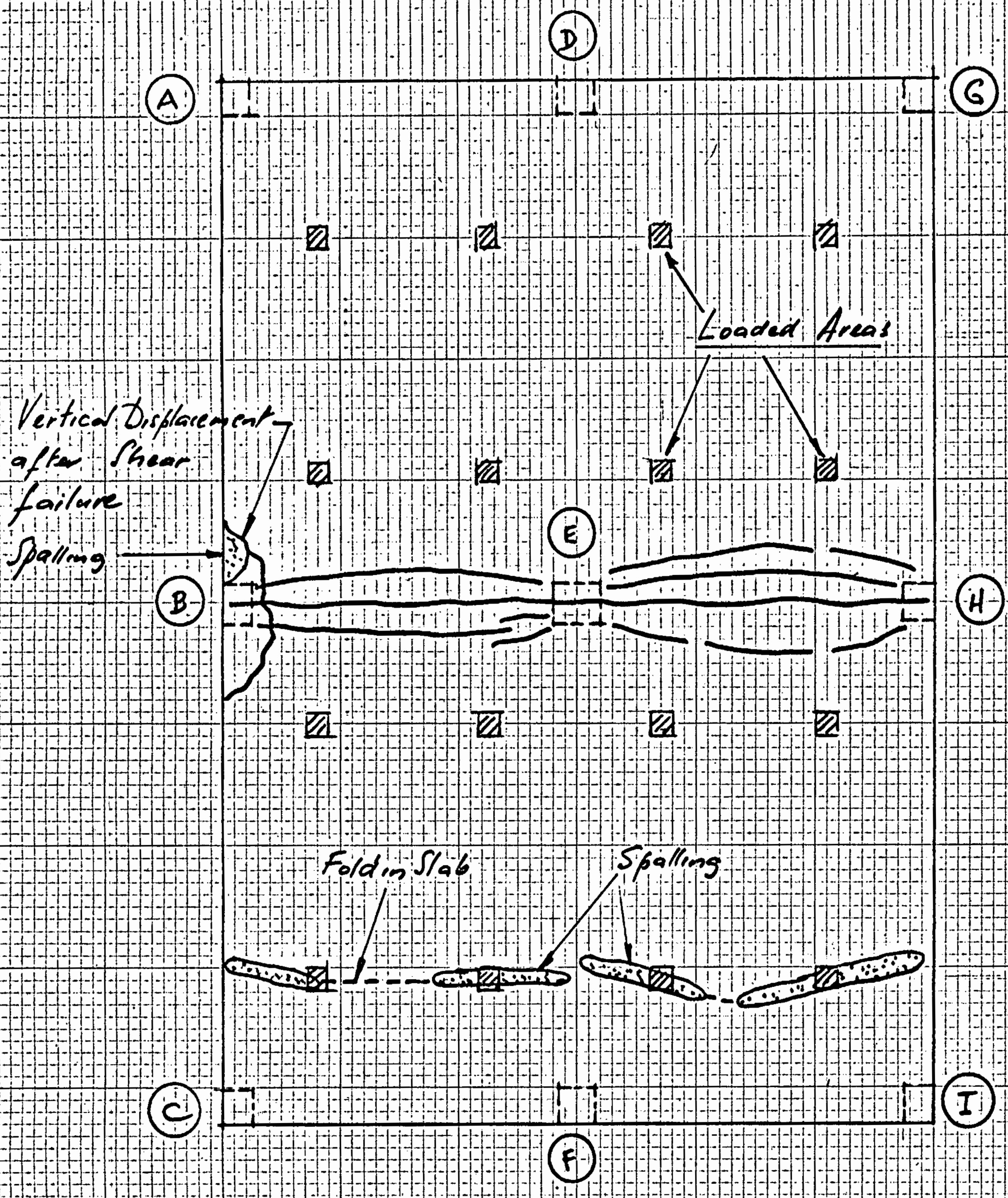


FIG 64 CRACK PATTERN IN TOP OF SLAB AT FAILURE

that the slab would behave elastically up to nearly the theoretical failure load.

8. PUNCHING STRENGTH

At column E the punching shear strength is given by various investigators as follows :-

A.C.I. $= v_u = 4 b_o d \sqrt{Fc'}$

$Fc' = 4{,}120$ p.s.i. $\sqrt{Fc'} = 0{,}45$ PMa $d = 43$ mm average

$V_u = 4 \times 4\ (150 + 43)\ 43 \times 0{,}45 = 60$ KN

Tasker : $V_u = bd\sqrt{Fc'} \left[2{,}5 + \frac{10}{(\frac{r}{d} + 1)}\right]$

$= 4 \times 150 \times 43 \times 0{,}45 \left[2{,}5 + \frac{10}{(\frac{150}{63} + 1)}\right]$

$= 64$ KN

At the Ultimate Load in bending, the reaction at column E was 50 + 5 = 55 KN which is less than the shear capacity.

9. ULTIMATE LOAD

From Yield-Line theory, a lower bound for the ultimate load Pu = 2P for flexural failure by the yield-line pattern in Figure 69.

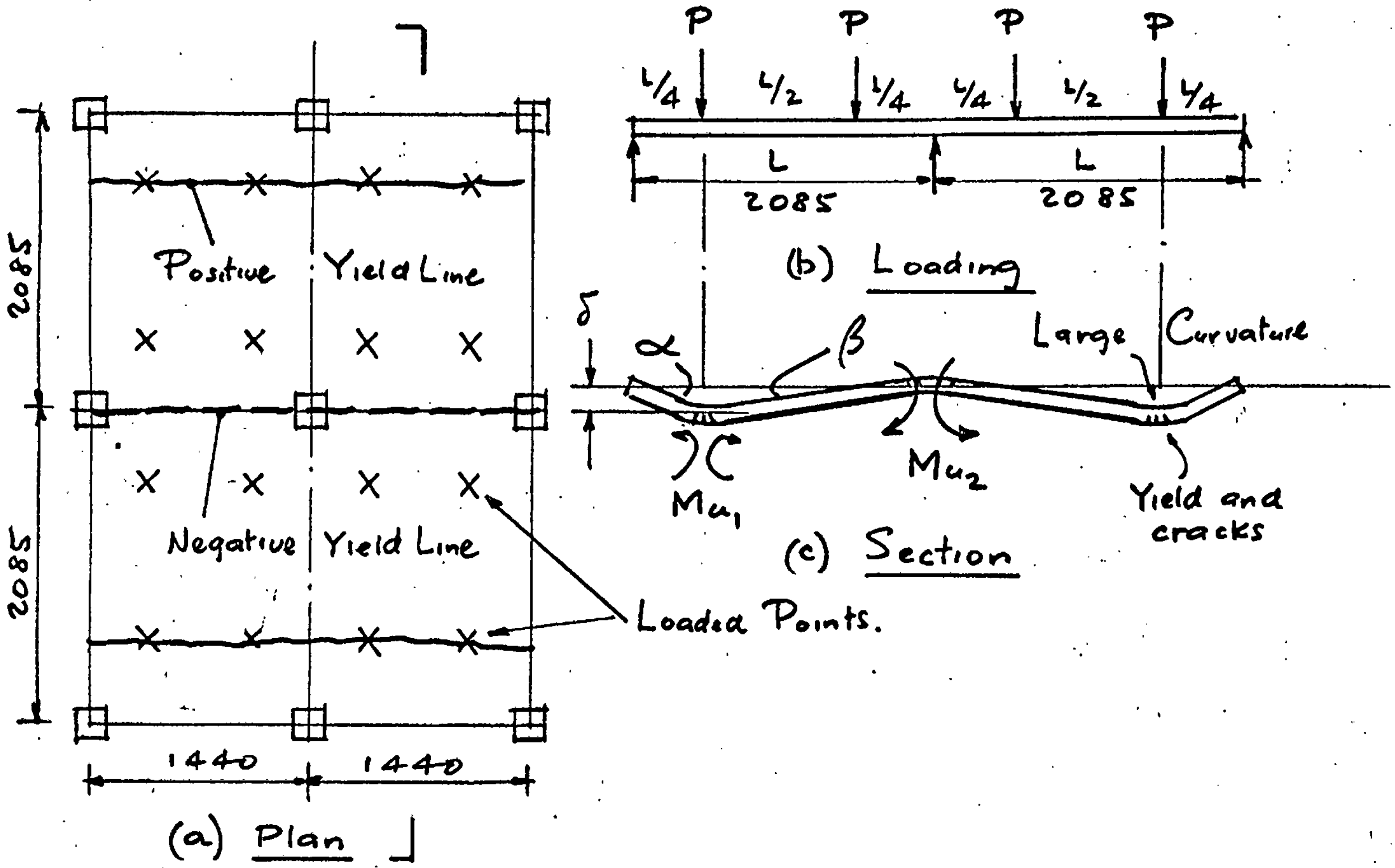


Fig. 69

is given by /

is given by the following Virtual-work equation :-

$$P\,\delta\,(1+\tfrac{1}{3}) + \tfrac{1}{2}W\delta = M_{u_1}(\alpha+\beta) + M_{u_2}\beta + 2\,\frac{Prb}{\pi}\beta$$

Where $\alpha = \delta/\frac{L}{4}$, $\beta = \delta/\frac{3L}{4}$

$$\text{Therefore } PL\left(1-\frac{2rb}{\pi}\right) = 4M_{u_1} + M_{u_2} - 3/_8\,WL \;\ldots\ldots.(69)$$

$+ As$ = 11 no. 8mm, $- As$ = 15 no. 8mm

$+ M_{u_1} \doteq 550 \times 360 \times 0{,}9 \times 50 \times 10^{-6} = +8{,}9$ KN.m

$- M_{u_2} \doteq 750 \times 360 \times 0{,}9 \times 46 \times 10^{-6} = 11{,}2$ KN.m

$P\,(2{,}085)\,(0{,}964) = 4 \times 8{,}9 + 11{,}2 - 3/_8 \times 4{,}5 \times 2{,}09$

$P = 21{,}5$ $\qquad P_u = 2P = 43$ KN ⟶

Actual superimposed load at failure was 49KN. The higher failure load may be due to membrane action after yield or the steel reaching a stress higher than its yield stress when the concrete failed in compression.

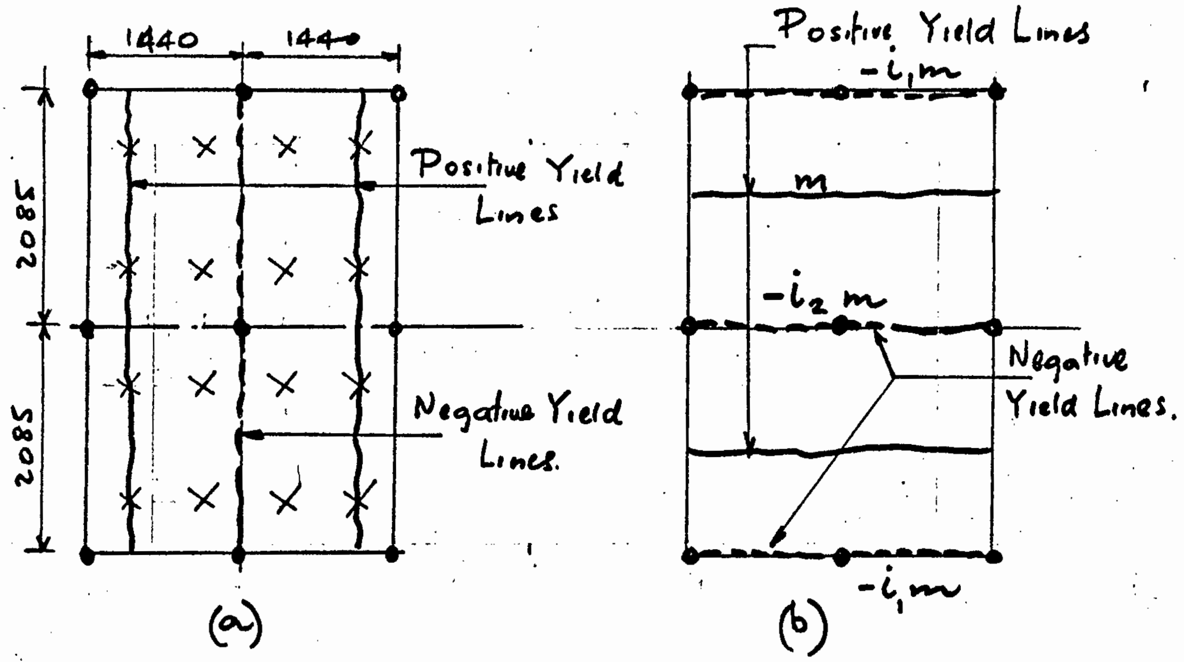


Fig. 70

For the Yield-Line pattern in Figure 70a the Ultimate Load is determined by a similar equation to (69) and is greater than 50KN.

$+M_{u_1} = 14 \times 28{,}3 \times 360 \times 0{,}9 \times 44 = +5{,}7$ KN.m

$-M_{u_2} = 12 \times 50 \times 360 \times 0{,}9 \times 40 = -7{,}8$ KN.m

This confirms the test result that failure will occur according to the yield-line pattern in Figure 69.

As a matter of /

As a matter of interest the Ultimate uniformly distributed panel load was calculated by the following formula (See Fig. 70 b)

(R33) $$m = \frac{WL^2}{2\left(\sqrt{1 + i_1} + \sqrt{1 + i_2}\right)^2}$$

where m = positive yield moment = 8,9 KN.m

$i_1 = 0$ $\quad i_2 = \frac{112}{8,9} = 1,25$ $\quad L = 2,085$

WL = Panel Load = 54KN including the dead load of the slab.

10. COMPUTED DEFLECTIONS

The deflections at the principal nodes for

(a) Concentrated load at the quarter-points of each panel

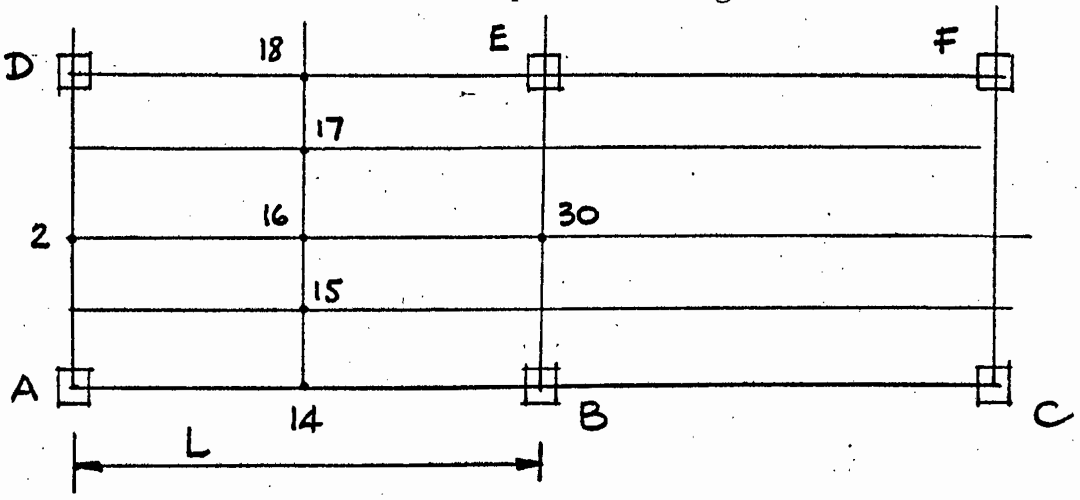
(b) Uniformly distributed load on each panel

are shown below in terms of $\frac{WL^2}{D}$

where W = Total load per panel

L = Long Span

D = Slab stiffness per unit length



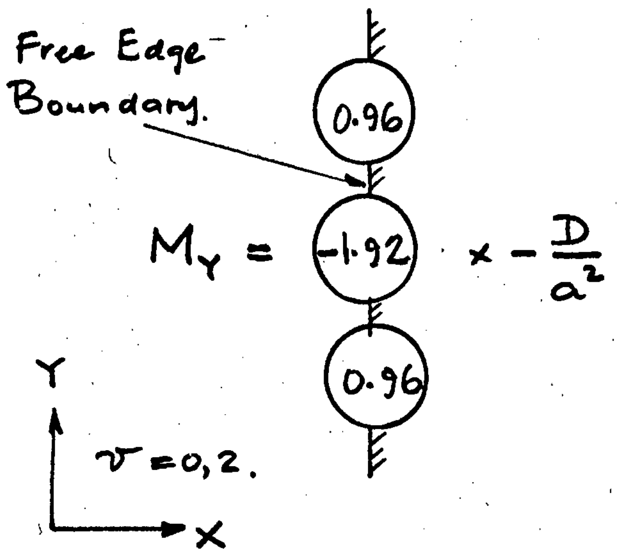
	(a) Quarter Point Loads	(b) Uniformly Distributed Load
Node	$\Delta = \alpha \frac{WL^2}{D}$	$= \beta \frac{WL^2}{D}$
	α	β
2	0.00337	0.00332
14	0.00865	0.00856
15	0.0105	0.00905
16	0.0114	0.0108
Max. 17 ←	0.0116	0.0108
18	0.0113	0.0107
30	0.00482	0.00446

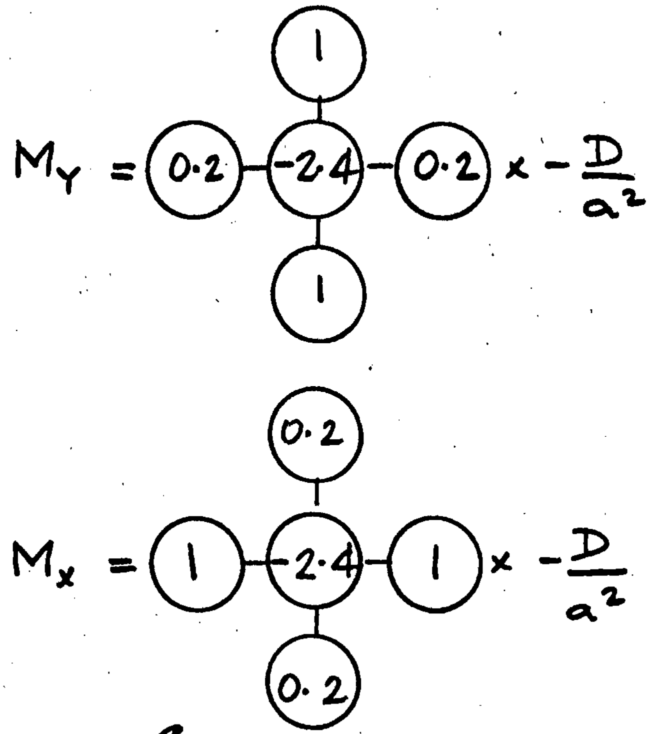
Contrary /

Contrary to expectation the results are in good agreement with a maximum error of 10%. However, it was surprising to find that the maximum deflection occurs at Node 17 which is off the centre of the Panel. The only possible explanation is that the relatively closer spacing of the columns in the short direction stiffens the slab so much that it approaches the behaviour of a slab on continuous supports along AD ..... BE ... and CF ,... In such a case, the maximum deflection would be at N18 and so in the plate the maximum deflection will be between the expected N16 and N18 i.e. towards N17 as the results show.

11. MOMENT VARIATIONS ACROSS THE PANEL WIDTH

From the Finite Difference Equations for the deflections at Nodes 1 - 31 due to a total of $8qa^2$ in each panel (Figure 48), the deflections were solved by the Computer in terms of qa^4/D. These are shown on page 124
For a panel load of 10 KN, $q = 10^{-2}$ N/mm^2 q = 354mm.
The Bending Moments M_x or M_y were then calculated by slide-rule, at the Nodes on the critical sections, using the following operators:-

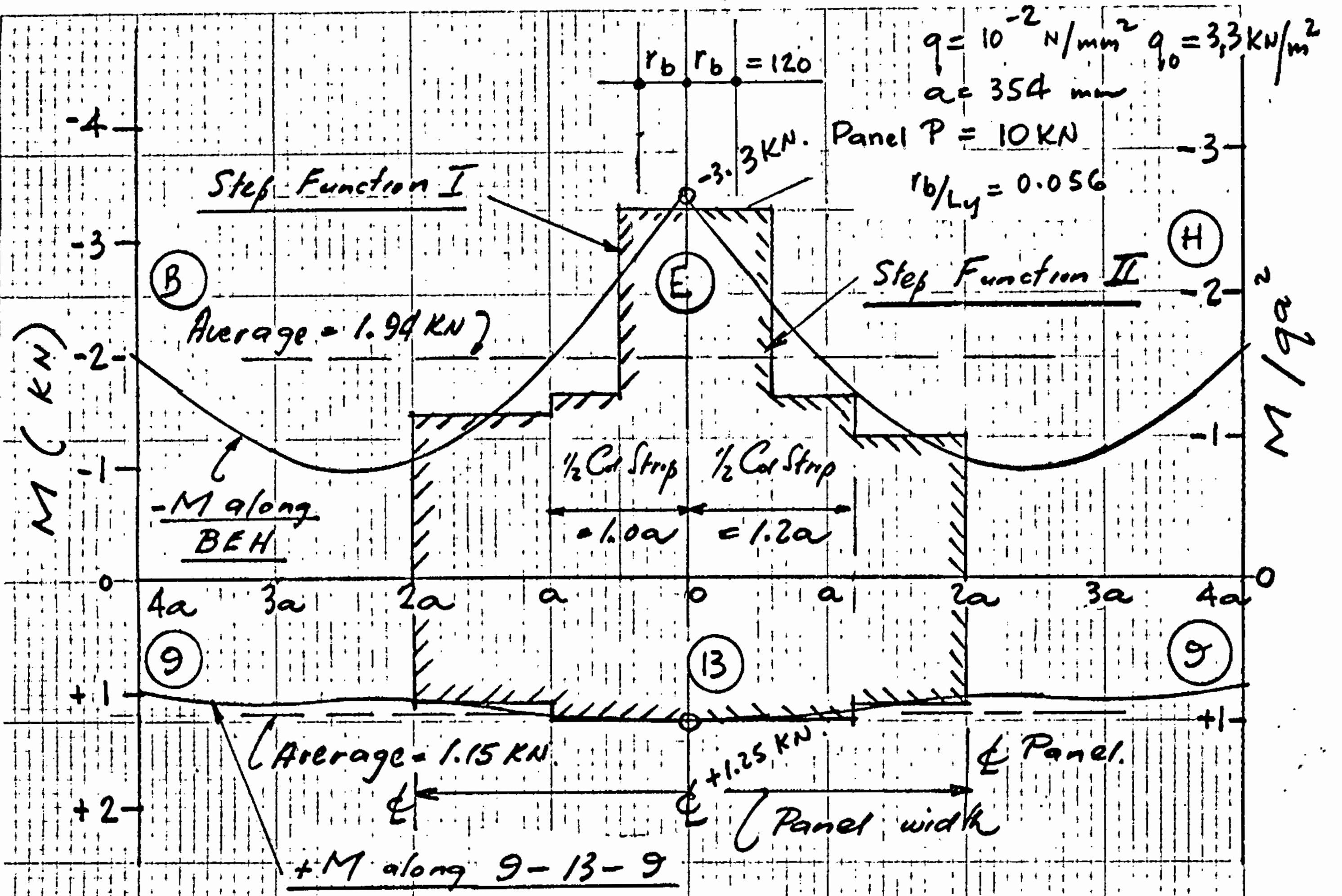




GENERAL OPERATORS.

The Bending Moments were plotted and smooth curves were drawn. (See Figures 71, 72). Positive and Negative moments My were then compared with the two alternative
(R. 4, 20) step-function approximations recommended in Section F.
For the two-span condition below the moments obtained (Fig.73) were +M = 0,18 PL
-M = 0,282 PL (no reduction)

By integrating /



I - From conventional width of Col. Strip = 2a and Tables AT2

II - From width of Col. Strip = $\frac{Lx.Ly}{Lx+Ly}$ = 2.4a and near conventional distribution. (Ref 4 - Fig 9)

FIG 71 - COMPARISON OF ELASTIC MOMENTS M_Y WITH STEP-FUNCTION APPROXIMATIONS.

NOTE: 1) Elastic Moment Distributions are calculated by Finite Differences.

2) Average Moments are calculated from conventional Elastic Analysis of a Frame of panel width.

3) Moments are Normal to the Planes shewn ie. M_y in Fig 48.

$\xi = 10^{-2}\ N/mm^2$
$q_0 = 33\ kN/m^2$ uniformly distributed
$a = 354$ mm

Panel Load $P = 10$ KN.

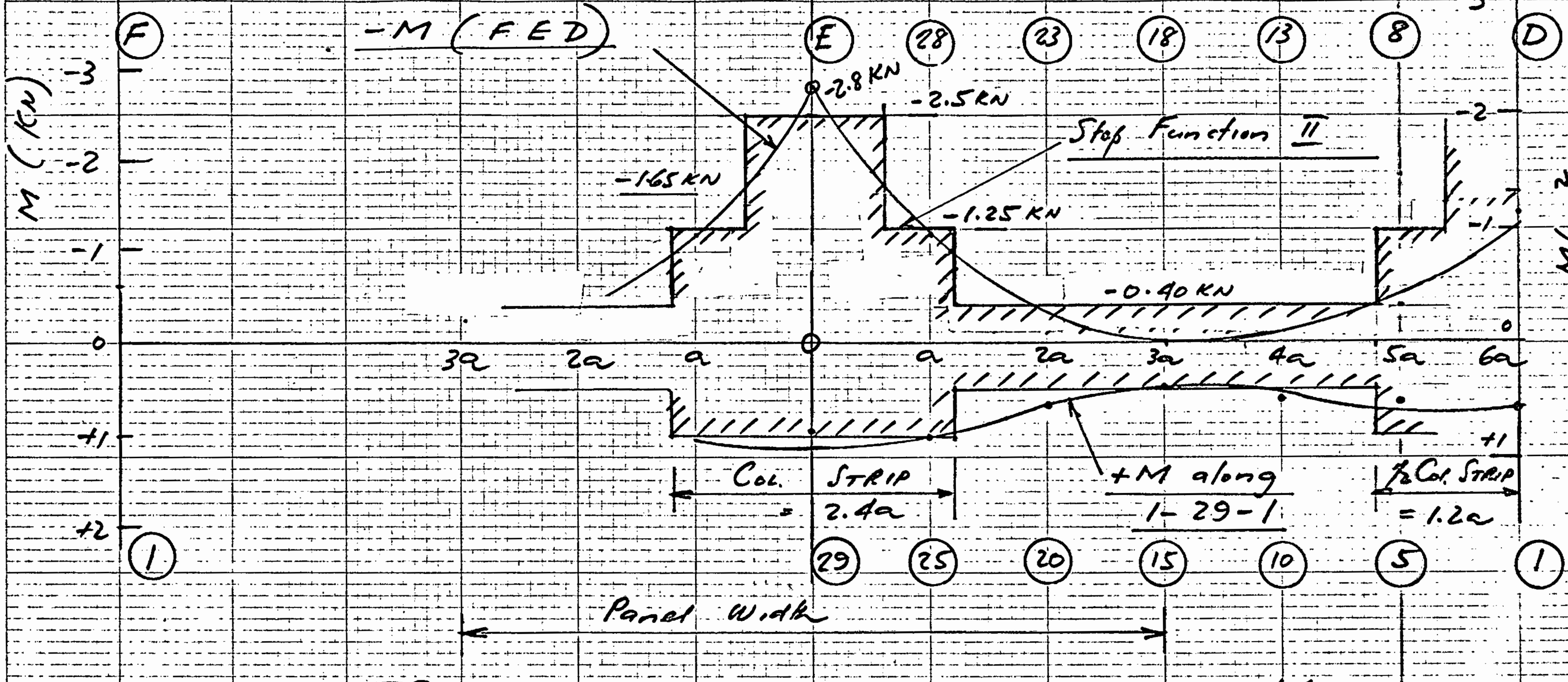
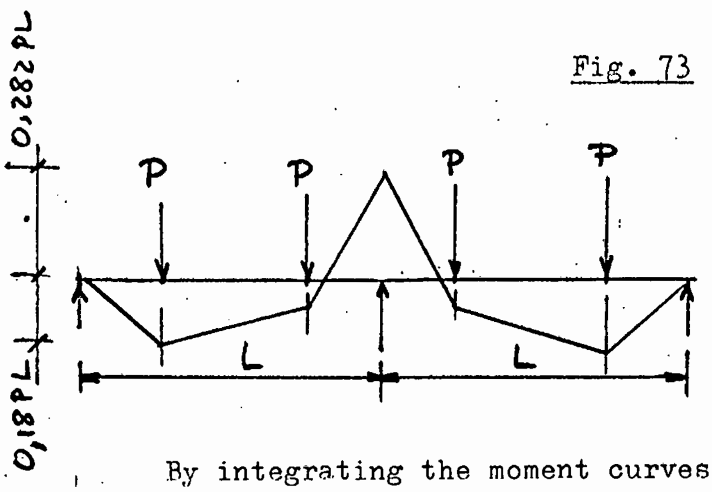


FIG. 72 - COMPARISON OF ELASTIC MOMENTS M_x WITH STEP FUNCTION APPROXIMATION

NOTE: i) Moments are Normal to the Planes shewn ie M_x in Fig 48.

Fig. 73



	Mx	My
+	1.27 KN.m	1.90
-	2.00	3.00

By integrating the moment curves and comparing with the total moments above, very good agreement was reached for the full panel width taken between the centre lines of the adjacent spans.

Within the Column strip, the maximum theoretical moments were only 10% greater than those assumed, but in the outer Column strip they increased to $\pm$ 40% above. Moments M_x were about double those assumed at the junction of the mid-strip and the column strip, but decreased to the same value within a short distance of $\pm \frac{1}{12}$ x panel width. The critical moments controlling slab behaviour are the negative moments. After reducing the theoretical moment by $\frac{Prb}{\pi}$ to allow for the effect of the column width, the resulting theoretical moment will invariably be less than that assumed.

The points plotted for positive M_x (in the short direction) along the critical section 1-29-1 did not lie on a smooth curve due to the moments in the column strip and so the best-fitting curve was drawn. No explanation could be found for this irregular moment distribution. It was also noticed that in all four theoretical distributions the moment at the centre column was much greater than at the edge column the discrepancy being larger in the case of the negative moment.

Briefly good agreement was obtained with step function II which is based upon a revised column strip width $\frac{L_x.L_y}{L_x+L_y}$ used in conjunction with conventional distribution between inner column strip, outer column strip and mid strip.

12. WALL BEHAVIOUR

The wall from G-H-I reinforced with "brick force" cracked away from the slab during the first loading sequence at the 20% level. The maximum crack width was then 0,75mm. The crack width increased to 5mm at the 60% level midway between H and I and was 1mm over support H. At supports I and G slight cracking at slab level/

level indicated some relative inward movement of the top of the slab due to its end rotations.

During the second loading sequence the maximum crack width started at $\pm$ 2mm and increased to $\pm$ 30mm at 90% of failure and 40mm at failure. Except for the opening of the wall to slab joint, no other cracks occurred in the wall G-H-I throughout the test.

The reinforced wall D-E-F, open-jointed at E, behaved in a similar fashion although the crack widths were smaller. Hair cracks commenced at the 30% level in the first loading and increased to 4mm at the 60% level.

The unreinforced walls A-B-C and D-E-F behaved slightly differently. During the first loading, hair cracks opened in A-B-C in the first mortar joint above the slab at the 30% level and increased to 3mm at the 60% level. Also at this level a smaller crack of $\pm$ 1mm opened at the slab level.

Wall D-E-F showed a 3mm crack in the slab joint and a 2mm crack in the joint | course above at the 60% level.

Wall G-A (unreinforced) showed a 0,5mm crack in the slab joint at the 60% level.

The behaviour of the unreinforced walls in the second loading test was similar to the first, but the walls had wider cracks at the same load levels.

Except for the horizontal cracks in the slab to wall joints and in the first mortar joints above, no further cracks were observed in these walls throughout either loading test.

After the formation of the above horizontal cracks, all the walls appeared to span independently of the slab between the supports, leading to the conclusion that the wall had negligible effect on the slab moments at the outset and no effect on the deflections or moments after the slab cracking load had been reached.

13. TEST SHORT-COMINGS

The loading system applied 4 concentrated loads at the panel quarter points to simulate a distributed load. As a result the total negative moments were $\frac{0,142}{0,125} = 13\%$ higher and the total positive moments were $\frac{0,18}{0,14} = 29\%$ higher than if the panel load was uniformly distributed.

These /

These discrepancies are too large to assume equivalence.

The best methods to simulate a distributed load are to place either

(a) A tank of water on top of the slab

(R.26,28) (b) Plastic bags filled with air between the slab and a supporting frame as has been done by other investigators.

Dial guages clamped to a steel frame suspended from the same steel girder as the hydraulic jacks, gave incorrect readings owing to the upward deflection of the girder, thus necessitating corrections. The dials could only be read from cramped positions on top of the slab and it is inevitable that some would be disturbed without our knowledge. Dials should preferably be placed on stands under the slab with sufficient head room to facilitate readings and the marking up of cracks.

The rubber pads under the slab were provided to permit slab rotations and a near-uniform distribution of pressure from the supports. Unfortunately the centre pad in particular compressed considerably (± 4mm at 80% level) although it may have been less if the readings had been corrected to allow for the girder's upward deflection. Steel

(R28) plates on roller bearings would have been ideal.

The dial guages used had ranges from 5mm to 20mm. At the 80% load level and above, most of the dials left the slab and no readings were possible. It is essential that the dials have ranges commensurate with the deflections expected.

Strain readings of the brick work were largely ineffective as the discs glued to the wall either moved during readings or fell off. However, no vertical cracks were observed during the tests and so readings were not necessary. For future tests, the discs should be glued to the brickwork on either side of vertical mortar joints with an epoxy at least 24 hours before the test.

14. GENERAL TEST CONCLUSIONS

Despite its shortcomings, the test enabled some useful conclusions to be drawn as follows:-

(1) The slab behaved elastically right up to ± 75% of the theoretical Ultimate Load.

(2) The Ultimate Load in bendings was approximately 14% greater than that predicted by the Yield-Line theory due possibly to strain-hardening of the steel or membrane

action or a combination of both.

(3) For loads below the cracking load, deflection could be predicted by the finite difference equations using the gross section stiffness to within 25% in all cases. For loads below $\frac{2}{3}$ of the cracking load, the deflections could be predicted to within 10%. The deflections throughout were less than if calculated using the cracked section stiffness for loads less than 75% of the theoretical ultimate, i.e. within the elastic range of the slab. The areas under the moment curves agreed well with the total moments across the panel width from elastic theory and we conclude that actual discrete moments though not measured, would agree with the theoretical for loads less than the cracking load. The step-function approximation to the actual moment-distribution curve was based on a revised width of column strip. It agreed reasonably well with the theoretical moments but produced local overstressing at design loads. In the actual test, no cracks were observed at loads below 90% of the theoretical ultimate and hence the approximation served its purpose.

(4) Solid walls of normal proportions (length:height 2,5:1) in line with the supports had negligible effect on the slab moments and deflections and conversely even large slab deflections from the walls produced no cracks in either reinforced or unreinforced walls above the first brick course. Similar walls in an actual slab would appear to require no special treatment to prevent cracks.

J. CONCLUSIONS

The optimum design of a flat concrete plate is that which ensures that its strength and behaviour satisfy the following criteria at the lowest relative cost.

1. (a) The Ultimate Load capacity before failure in bending or shear should be not less than the total design load multiplied by an appropriate Load Factor.

   (b) It should behave elastically for any live load less than twice the design live load but not for more than 80% of the Ultimate Superimposed Load i.e. retain no permanent deflection after unloading.

   (c) At working load, cracking, if any, should not be visible to the naked eye

   (d) Deflections due to the transient live-load should be a practical minimum.

   (e) Creep and shrinkage deflections due to the dead load and sustained live-loads should not exceed $\frac{L}{720}$ or 8 mm if it supports brick partitions and $\frac{L}{540}$ for all other loads.

2. The relative cost of the slab is the total cost of steel and concrete per square metre. If criterion 1(a) only is satisfied for minimum cost, the slab depth will be much less than if all the criteria are satisfied, but the relative costs should not differ by more than 10%. In terms of the building cost as a whole this represents an increase of about 1% if the slabs are "heavy" and less for "light" slabs.

3. The Elastic, Empirical and Yield Line methods of design satisfy only criterion 1(a) fully and this has been amply demonstrated by many tests and structures in use. The slabs designed are roughly equivalent in cost as well as performance. The limitation of the span to slab depth ratio is by itself insufficient to control deflection and cracking behaviour and must be coupled to a limitation of concrete strains at mid-span.

4. The Finite Difference method enables all the criteria to be satisfied directly provided mid-span cracking is avoided. However, the writer feels that more time is required by the designer for the various operations such as writing and checking the equations, calculating moments from the print-out of deflections, plotting the graphs and detailing the reinforcement than in any other method. Furthermore, no allowance

can be /

can be directly made of the effect of column stiffnesses. As other methods are possible which can satisfy all the criteria the writer cannot justify its use for rectangular plates.

5. The Elastic-Plastic Method using unstressed reinforcement differs from the Elastic in the distribution of moments only, the cost being the same. It satisfies criteria 1(a), 1(b) and 1(d) but ignores the others. However, they can also be satisfied by the following additional procedure :-

   (a) Determine the slab depth so that the cracking moment across the full panel width at mid-span is equal to the positive moment due to the sustained loads.

   (b) Check the depth for punching shear by the formulae recommended by Moe, Tasker and Wyatt or the A.C.I. Code for shear. If necessary structural steel shear heads can be used to spread the column reaction into the slab.

   (c) Check the Elastic deflection due to the sustained loads and hence the ultimate creep deflection which is twice the Elastic deflection, by the following approximate formula using the gross section stiffness

$$\frac{\Delta}{L} = \frac{L}{48EI}\,(5M_1 - M_2) \quad \text{where}$$

Δ = deflection at mid point between two columns
L = Long Span
M_1 = Positive moment in the column strip
M_2 = Negative " " " " "
E_c = Instantaneous Elastic Modulus of the concrete
I = Gross Moment of inertia of the column strip

Alternatively the deflection can be indirectly controlled by making $\frac{L}{d} \leq 32$ for all spans where L = Longer span

6. The writer believes that the slab deflections which may affect brick partitions are the deflections in the plane of the wall, of the panel centre in relation to its ends, and <u>not</u> of the panel centre point in relation to its column supports which is slightly greater depending on the aspect ratio. In the tests the walls were able to "arch" between columns independently of the slab without distress to itself or affecting the slab moments or behaviour.

   The effect of walls partly interrupted by openings in random positions on the slab was not investigated and could

be the /

be the subject of further research. Light brick reinforcement will considerably improve their ability to withstand uneven floor deflection or movements without the sudden formation of large cracks.

Distress to brick partitions may be due mainly to the additional load transferred from higher slabs under their creep deflections. To reduce this possibility it is proposed that

(a) building of walls be delayed as long as possible
(b) walls should not be built hard up against the underside of the slab above but a gap of at least 10 mm should be provided.
(c) creep deflections of all slabs can be reduced by better curing, longer propping through at least two floors and by reducing water content of the mixes to a minimum. Apart from reducing the possibility of unsightly cracks, maintenance costs will be substantially reduced.

7. The limitation of the Elastic behaviour in criterion 1(b) will permit Plastic deformation before collapse for a limited range of loading which would give warning of serious overloads.

8. By pre-stressing the plate in both directions according to the Elastic-Plastic method, all criteria are satisfied. Further advantages gained are as follows:

   (a) Short-term deflection is zero at sustained loads and long-term deflections are a minimum

   (b) The slab is crack-free and water-tight due to the uniform compression, brick partitions are crack-free and overall maintenance is therefore minimised.

9. The tests carried out on the Experimental Model which was designed by the Elastic-Plastic method confirmed the expected behaviour of such slabs. The failure load was, however, 14% higher than predicted but this may be due to uncertainty regarding the onset of yield in the steel, which may have possibly commenced later than assumed owing to the large variation in the commencement of yield which is possible in mild steel.

10. Optimum design can, as illustrated, be achieved in the Drawing Office but the slab behaviour will not be optimum unless there is full co-operation from the Contractor in ensuring that the Engineer's carefully prepared specification is carried out to the letter to ensure proper control of the quantity of water, compaction of concrete, maintenance of steel in the right position, curing and the propping of floors.

Provided /

Provided this is done, there can be no reason why optimum behaviour of flat rectangular concrete plates cannot be achieved economically.

11. The behaviour of the slab at working loads is largely dependent upon both its tensile strength and the tensile cracking strain. Consequently cylinder splitting tests to measure and control the concrete tensile strength of flat plates should be made in addition to crushing tests. Further research on measures or additives likely to increase the tensile strength and decrease the tensile cracking strain of concrete would be a step nearer the optimum design of even more slender flat plates.

ACKNOWLEDGEMENTS

The Writer hereby wishes to record his sincere thanks to the Civil Engineering Faculty of the U.C.T. which placed all the facilities of its laboratory at his disposal, to Mr. W. S. Doyle, Lecturer in Structures at U.C.T. for his guidance and assistance in preparing the experiment and to the staff of the laboratory who so ably and enthusiastically built the test slab.

PART 3. REFERENCES

BROTCHIE J.F.

1. General Method of Analysis of Flat Slabs and Plates A.C.I. July 1957.
2. Elastic-Plastic Analysis of Transversely Loaded Plates. Trans. A.S.C.E. October 1960.
3. A Concept for Direct Design of Structures. Trans. I.E. Australia. September 1963.
4. Flat Plate Structures Elastic-Plastic Analysis A.C.I. August 1964.
5. A Refined Theory for R.C. Slabs. Journal I.E. Australia. October 1963.
6. Direct Design of Prestressed Concrete Flat Plate Structures. (Constr. Review. January 1964)
7. On Moments in Slabs (Constr. Review. March 1967)
8. Experimental Study of a Prestressed Concrete Flat Plate Structure. (Trans. Inst. of Eng. Australia) October 1967.
9. A Criterion for Optimal Design of Plates (A.C.I. November 1969)

F.A. BLAKEY

10. Deformations of Experimental Lightweight Flat Plate Structures. (Trans. Inst. of Eng. Australia. March 1961)
11. Design of Flat Plates by Simple Analysis (Constr. Review November 1962)
12. Australian Experiments with Flat Plates. A.C.I. April 1963.
13. Deflection of Flat Plate Structures. Civil Eng. V.58 1963.
14. Deflection as a Design Criterion in Concrete Buildings (Trans. Inst. E. Australia. September 1963)
15. Towards an Australian Structural Form - The Flat Plate (Architect in Australia. September 1965)
16. Changing Fashions in Concrete. Australian Bldg. Science. December 1965.
17. An Experimental Study of Flat Plates. C.I.B. No. 4 1965.
18. A Review of Some Experimental Studies of Slab-Column Junction. Constr. Review July 1967.
19. Lightweight Aggregate Concrete in Flat Plate Floors C.S.I.R.O. V.II. - 1968.

J.J. RUSSELL

20. Analytical Data for Direct Design of Reinforcement Distribution in Internal Panels of Flat Plates. C.S.I.R.O. Australia. 1966.
21. Numerical Methods of Analysis by Computer. Civil Eng. V.58. 1963.

BERESFORD 22. Tests of Edge Column Connections of a Flat Plate Structure. Trans. Inst. of E. Australia. October 1967.

23. Analytical Examination of Propped Floors in Multi-Storey Construction. (Constr. Review. November 1964)

24. Experimental Lightweight Flat Plate Structure. C.S.I.R.O. Australia

Part I. Measurements and Observations during Erections-1961

II. Deformation due to self weight - 1961

III. Long Term Deformations - 1961

IV. Design and Erection of Structure with Concrete Columns - 1962.

V. Deformation under Lateral Load - 1962

VI. Design and Erection of Post Tensioned Flat Plate 1963.

VII. A Test to Destruction - 1963.

VIII. Test to failure of Prestressed Slabs (Gamble 1964)

25. Shear in Flat Plate Construction under Uniform Loading (Commonwealth Experimental Building Station-Tasker & Wyatt 1963)

26. Experimental Investigation of the Strength Behaviour of Prestressed Plate Div. of Bldg. Research. Gamble 1964)

27. Flat Slab solved by Model Analysis - Bowen & Staffer. A.C.I. (February 1955)

28. Strength of a Concrete Slab prestressed in Two Directions (Scordelis, Pister & Lin. A.C.I. September 1956)

29. Load Test on Flat Slab floor with Steel Grillage Caps (Meisel, Jensen, Wheeler. A.C.I. July 1958)

30. Ultimate Shear Strength of R.C. Flat Slabs, footings etc. without Shear Reinforcement (Whitney. A.C.I. October 1957)

31. Deflection of Multiple-Panel R.C. Floor Slabs (Vanderbilt, Sozen & Siess). Proc. A.S.C.E. - Structural Division. August 1965.

32. Design of Prestressed Concrete Structures. T.Y. Lin

33. Yield-line Theory for Ultimate Flexural Strength of R.C. Slabs. E. Hognestad. A.C.I. March 1953.

34. Theory of Plates & Shells. - Timoshenko, Woinowsky & Kruger. 1953.

35. Plastic & Elastic Design of Plates and Shells. Wood & Jones.

36. Reinforced Masonry Design & Practice. Proc. A.S.C.E. December 1961.

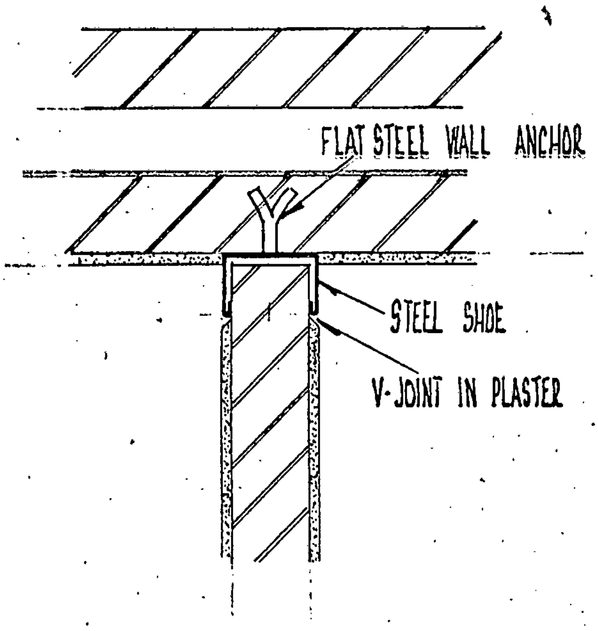
37. The Interaction of Floors and Beams. National Bldg. Studies. Research Paper No. 22. - Wood

38. European Committee for Concrete-Information Bulletin No.35.

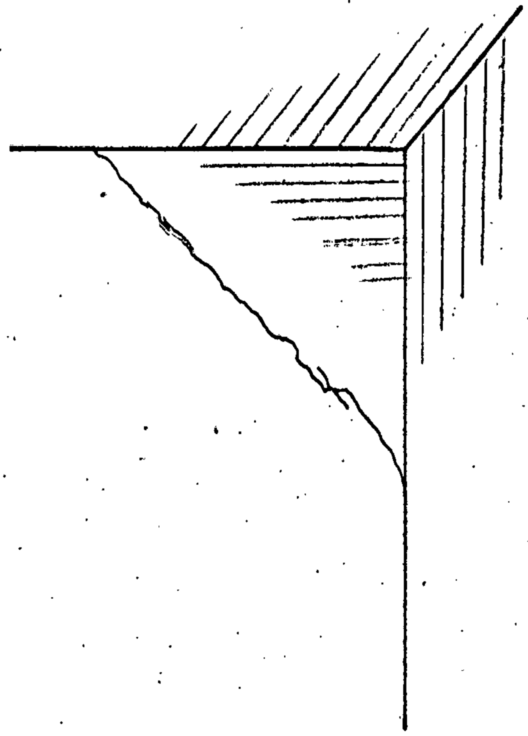
39. A.C.I. Manual of Concrete Practice

40. Sudden Collapse of Unbonded Prestressed Structures G. Rozvanny & J. Wood. A.C.I. February 1969.

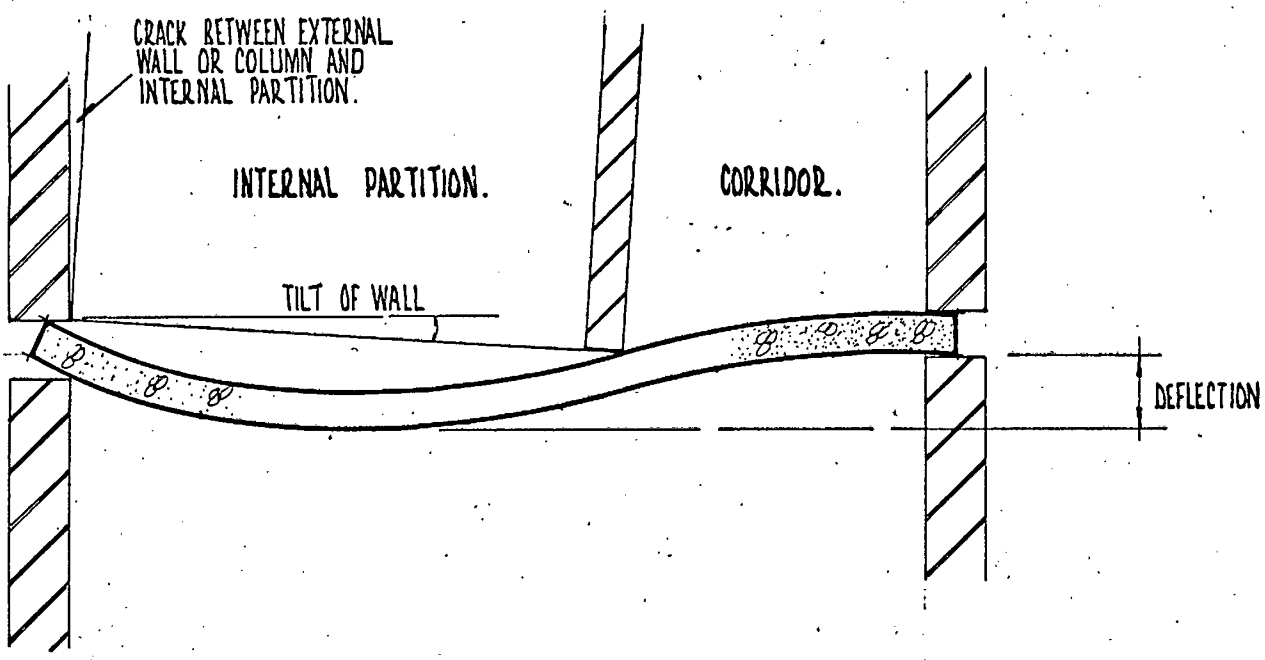
A P P E N D I X



MOVEMENT JOINT.

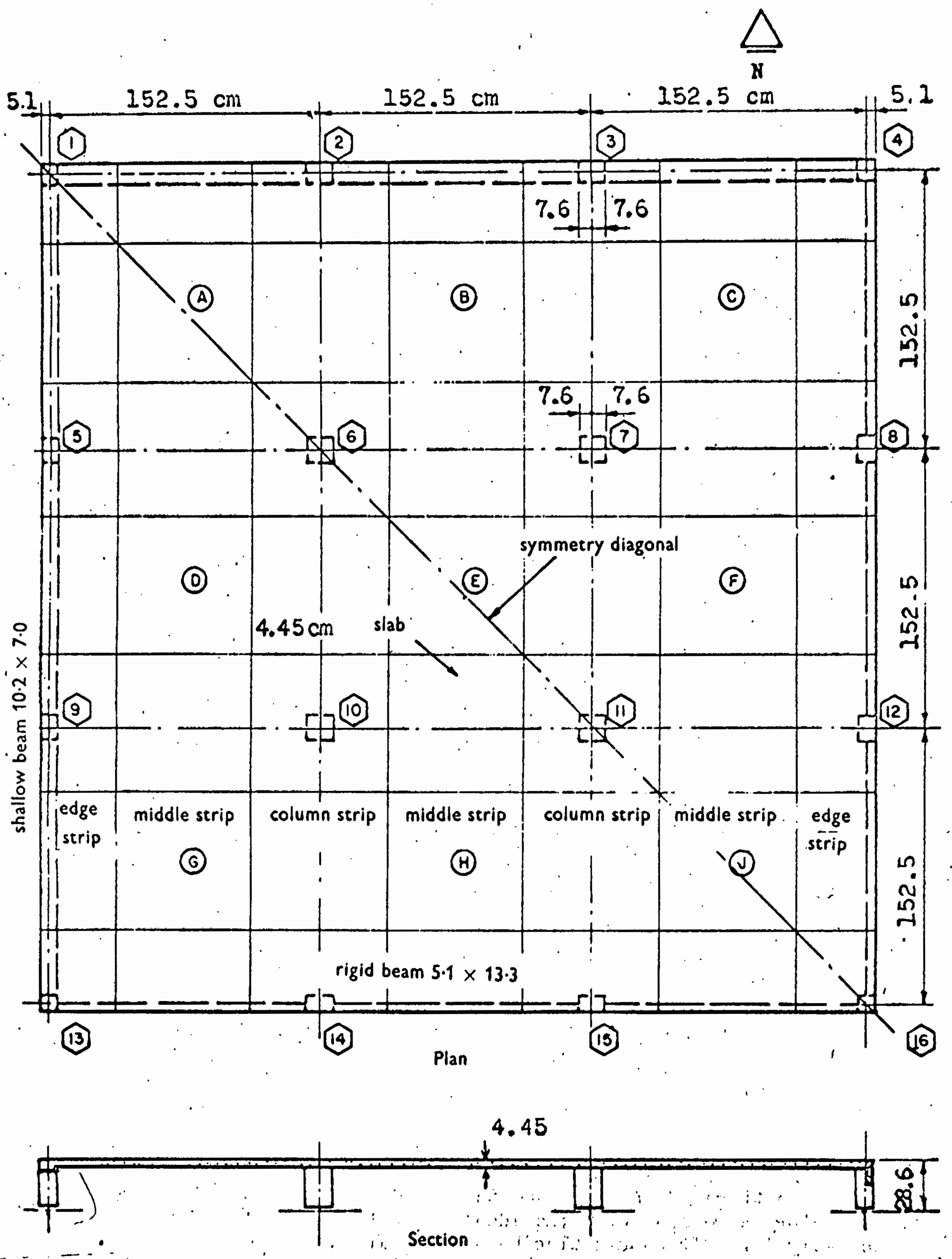


WALL CRACK DUE TO DEFECTION OF FLOOR BENEATH THE WALL.



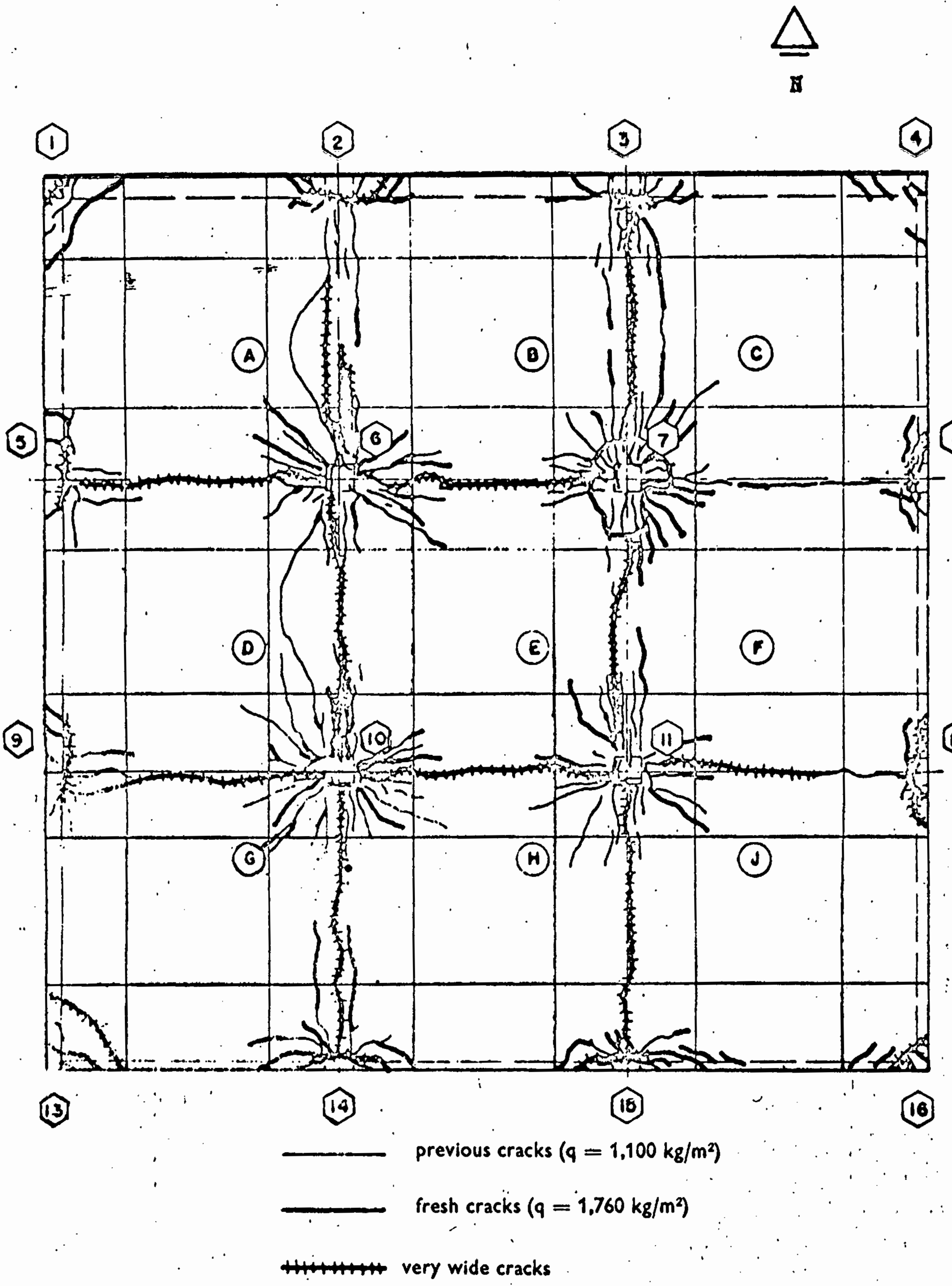
CRACKING CAUSED BY DEFLECTION OF FLAT PLATE.

FIG AF(1) - TYPICAL WALL CRACKS



The test slab.

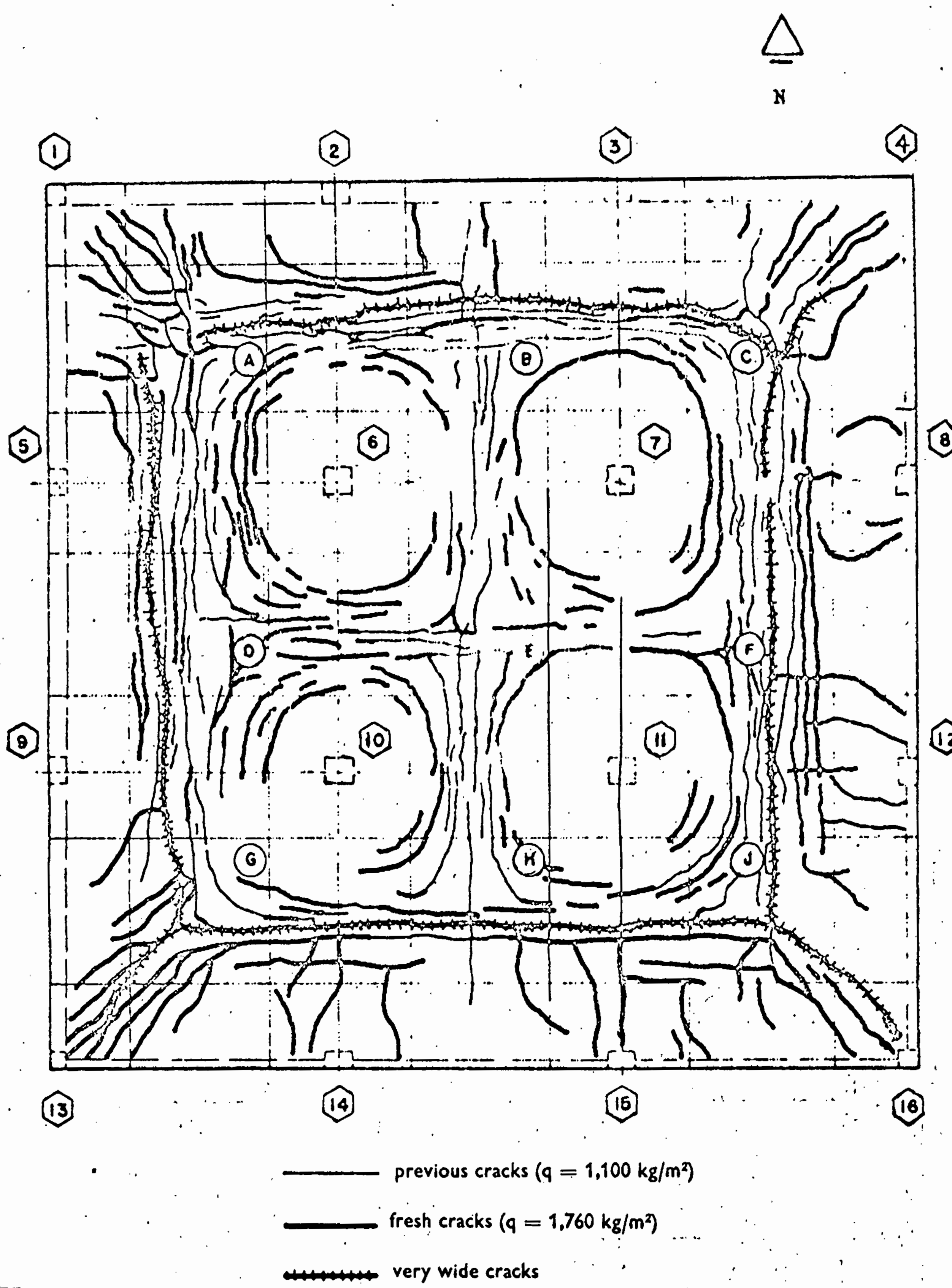
Fig. A.F2



Pattern of cracking on the top surface.

$q = 1,760$ kg/m^2

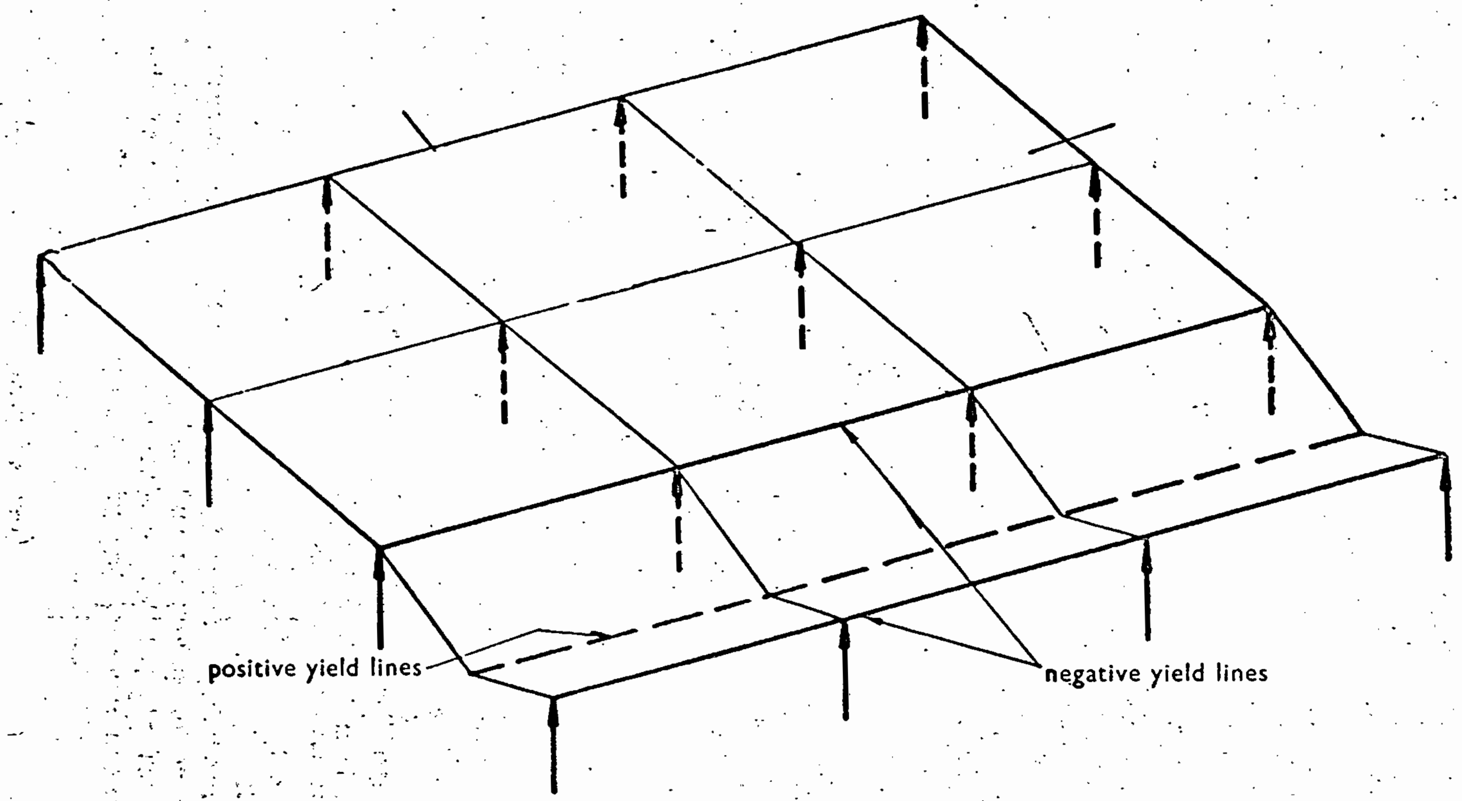
Fig. A.F3



Pattern of cracking on the bottom surface.

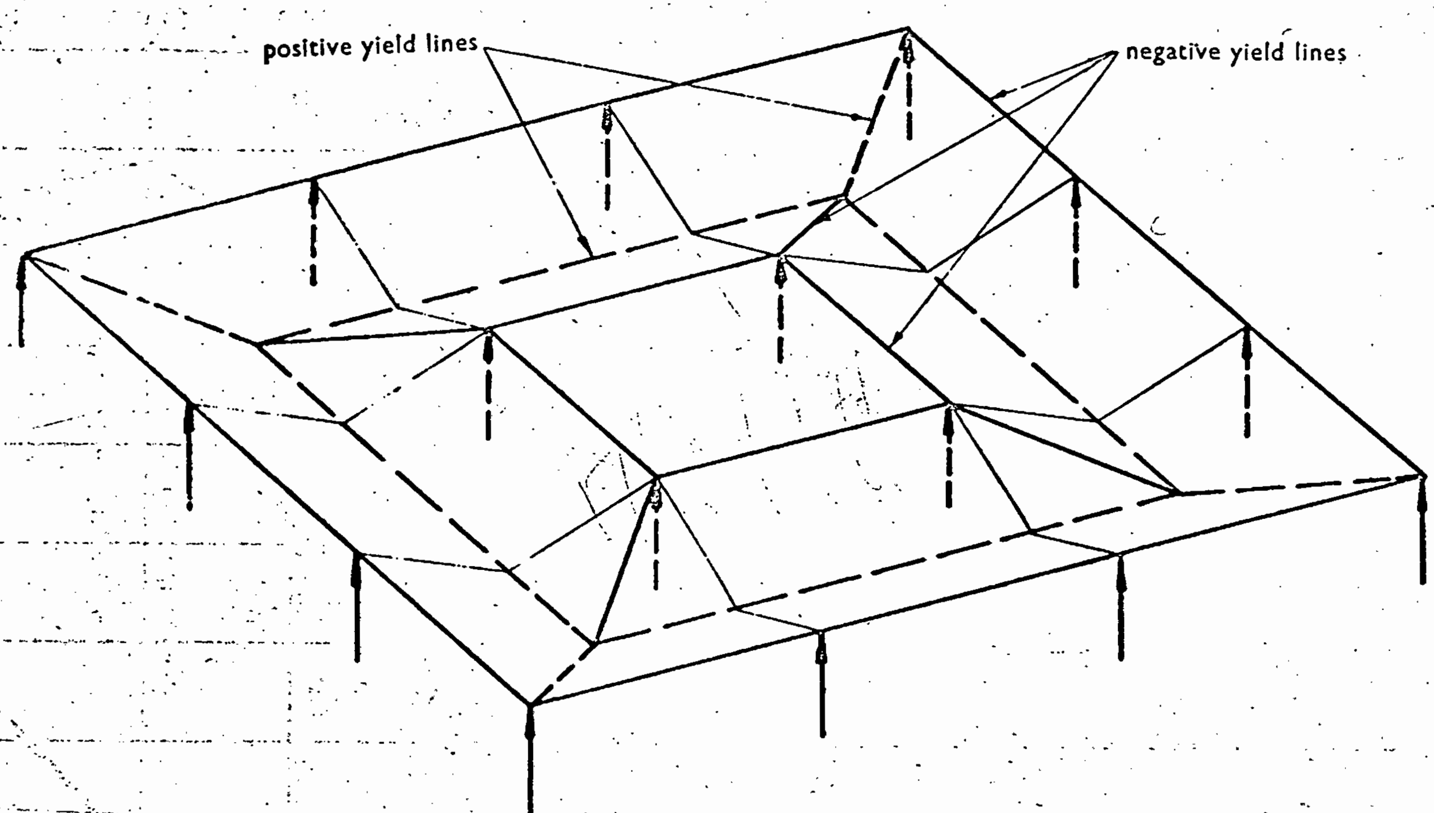
$q = 1,760\ kg/m^2$

Fig. A.F4



Fracture pattern, type 2.

Fig. A.F5



Fracture pattern, type 1.

Fig. A.F6

REINFORCEMENT DISTRIBUTION IN INTERNAL PANELS

(Mo)

DIVISION OF MOMENTS IN A SQUARE OR RECTANGULAR PANEL

L = slab span; r_b = effective column radius

Panel Shape $L_y : L_x$	Ratio r_b/L_y	Direction of Span	Negative Moment		Positive Moment	
			** Column Strip (%)	Middle Strip (%)	Column Strip (%)	Middle Strip (%)
1 : 1	0		50	16	21	13
	0·025		48	16	22	14
	0·05		45	17	23	15
	0·075		42	18	25	15
	0·1		39	18	26	17
1·1 : 1	0·025	X	49	15	23	13
		Y	47	18	20	15
	0·05	X	45	15	25	15
		Y	44	19	22	15
	0·075	X	42	16	28	14
		Y	40	20	24	16
1·2 : 1	0·025	X	50	13	25	12
		Y	46	19	20	15
	0·05	X	46	14	27	13
		Y	42	20	22	16
	0·075	X	42	14	30	14
		Y	39	21	23	17
1·3 : 1	0·025	X	52	11	26	11
		Y	44	20	20	16
	0·05	X	48	12	28	12
		Y	41	21	21	17
	0·075	X	44	13	31	12
		Y	38	22	22	18
1·5 : 1	0·025	X	54	9	29	8
		Y	42	22	19	17
	0·05	X	50	9	32	9
		Y	39	23	20	18
	0·075	X	45	10	35	10
		Y	36	24	21	19
2 : 1	0·025	X	55	5	35	5
		Y	39	26	18	17
	0·05	X	50	5	39	6
		Y	36	27	19	18
3 : 1	0·025	X	56	2	39	3
		Y	34	31	18	17
	0·05	X	47	2	46	5
		Y	32	31	19	18

** Width of column strip = width of mid strip = ½ panel width

Table A.T1

REINFORCEMENT DISTRIBUTION IN INTERNAL PANELS

DISTRIBUTION OF MOMENT ACROSS PANEL WIDTH

Panel Shape $L_y : L_x$	Ratio r_b/L_y	Direction of Span	Negative Moment		Positive Moment		Concentration over Column
			Column ** Strip (av.) (%)	Middle Strip (%)	Column Strip (%)	Middle Strip (%)	$\frac{p_{max}}{p_{av}}$
1 : 1	0		76	24	62	38	
	0·025		75	25	62	38	1·63
	0·05		72	28	62	38	1·35
	0·075		70	30	61	39	1·20
	0·1		68	32	61	39	1·12
1·1 : 1	0·025	*X*	78	22	63	37	1·73
		Y	73	27	58	42	1·58
	0·05	*X*	75	25	63	37	1·40
		Y	70	30	58	42	1·30
	0·075	*X*	72	28	64	36	1·23
		Y	68	32	59	41	1·15
1·2 : 1	0·025	*X*	80	20	68	32	1·77
		Y	71	29	57	43	1·53
	0·05	*X*	77	23	67	33	1·41
		Y	68	32	57	43	1·26
	0·075	*X*	75	25	67	33	1·22
		Y	65	35	57	43	1·13
1·3 : 1	0·025	*X*	82	18	71	29	1·83
		Y	69	32	55	45	1·45
	0·05	*X*	80	20	71	29	1·46
		Y	66	34	55	45	1·22
	0·075	*X*	77	23	71	29	1·28
		Y	63	37	55	45	1·11
1·5 : 1	0·025	*X*	86	14	77	23	1·90
		Y	65	35	53	47	1·36
	0·05	*X*	84	16	77	23	1·48
		Y	62	38	53	47	1·15
	0·075	*X*	81	19	77	23	1·29
		Y	60	40	53	47	1·06
2 : 1	0·025	*X*	92	8	87	13	2·13
		Y	60	40	51	49	1·21
	0·05	*X*	91	9	86	14	1·63
		Y	57	43	51	49	1·06
3 : 1	0·025	*X*	97	3	93	7	2·49
		Y	55	45	50	50	1·08
	0·05	*X*	96	4	90	10	1·78
		Y	53	47	50	50	1·02
Code requirement (see Table 1)			76	24	60	40	1·0†

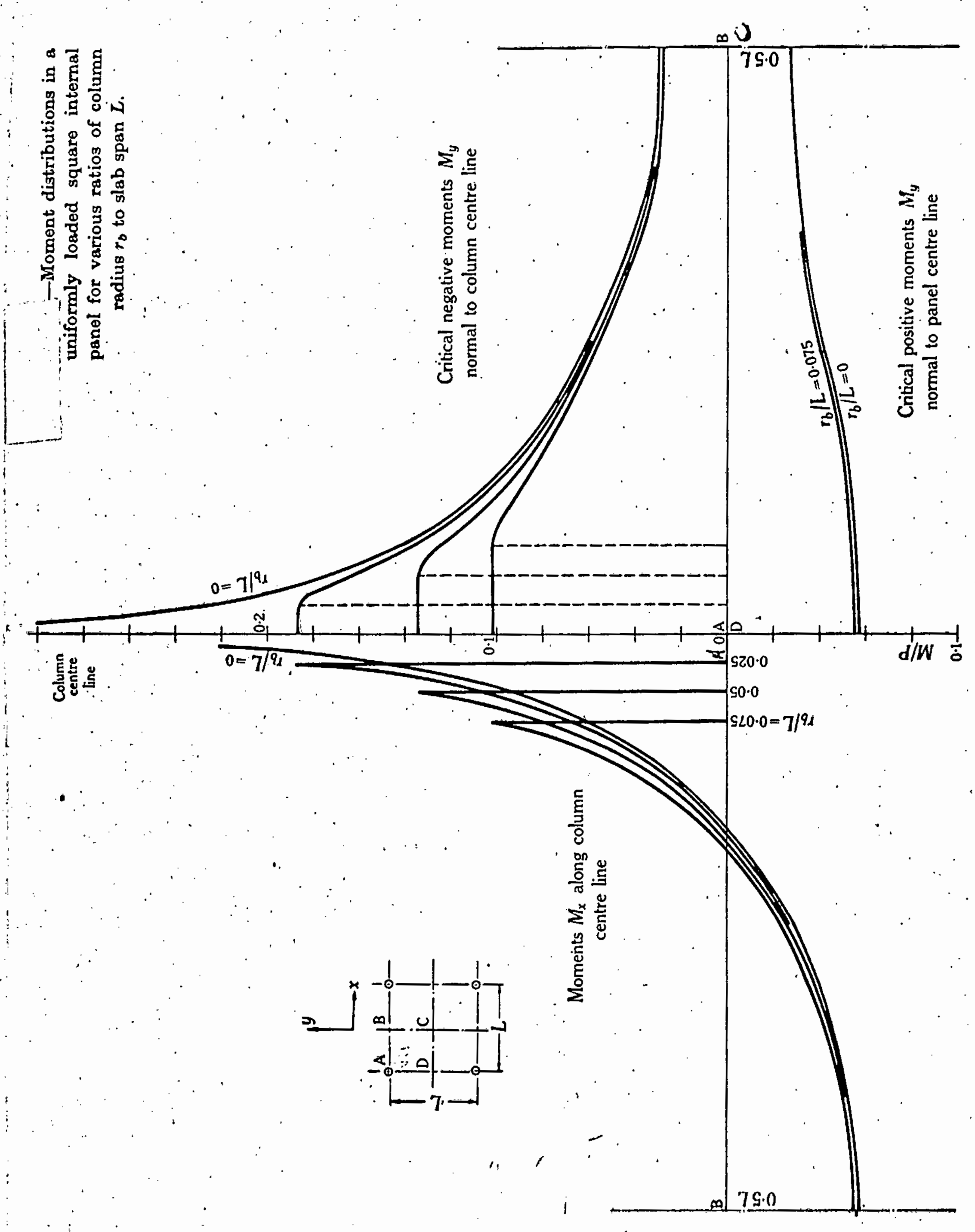
* Ratio of steel percentage per unit width over column to average percentage of top steel in column strip.

† 25% of negative steel within the column strip must be located within the width $2r_a + 2t$.

** Width of Column Strip = width of Mid. Strip = ½ of Panel Width.

Table A.T2

REINFORCEMENT DISTRIBUTION IN INTERNAL PANELS



—Moment distributions in a uniformly loaded square internal panel for various ratios of column radius r_b to slab span L.

Fig. A.F7

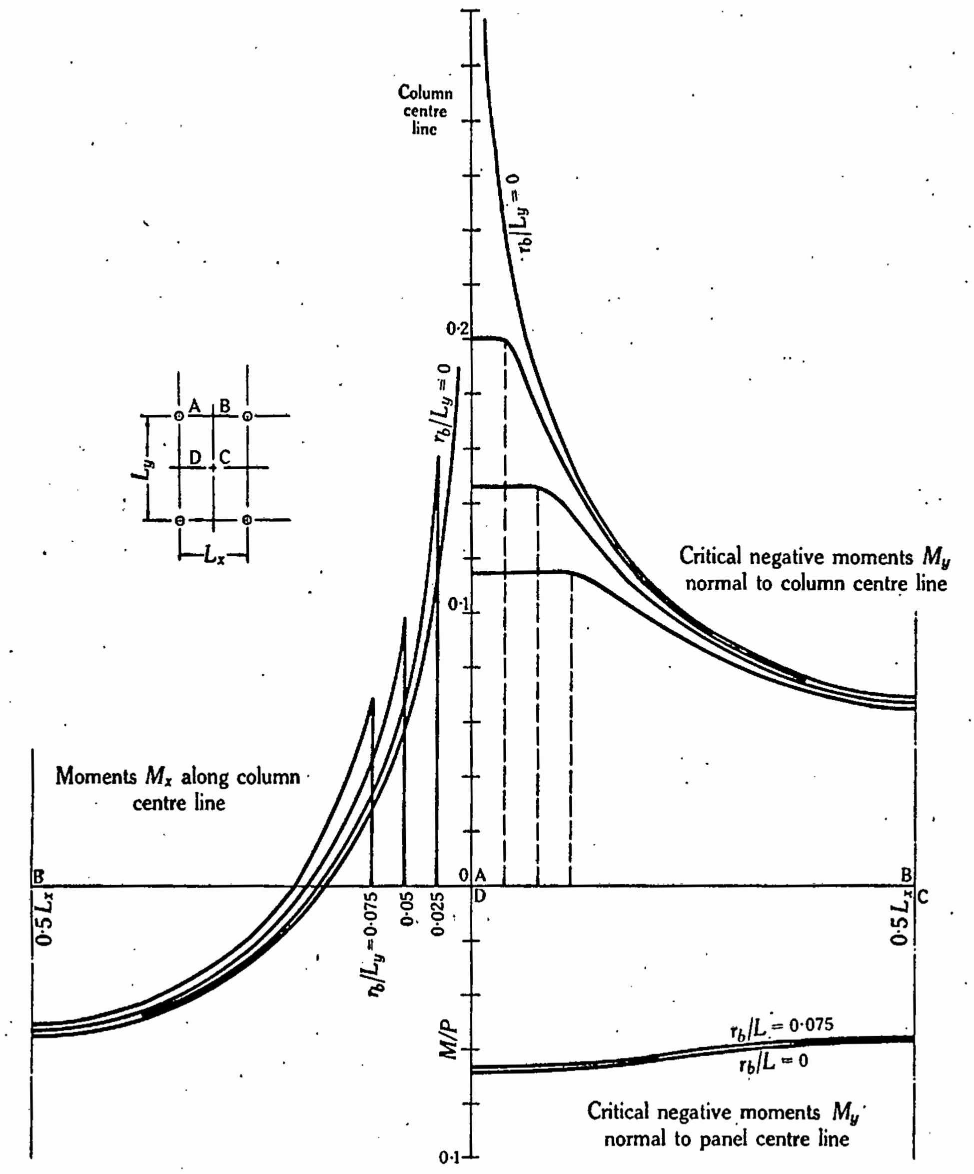


Fig. —Moment distributions over width L_x for rectangular panel ($L_y = 1\cdot5\ L_x$) under uniform loading (cf. Fig. 16).

The appropriate step functions under this theoretical treatment were obtained from numerical integration of the curves. These numerical integrations were carried out on a National Elliott 803 computer for both the positive and negative moment curves.

REINFORCEMENT DISTRIBUTION IN INTERNAL PANELS

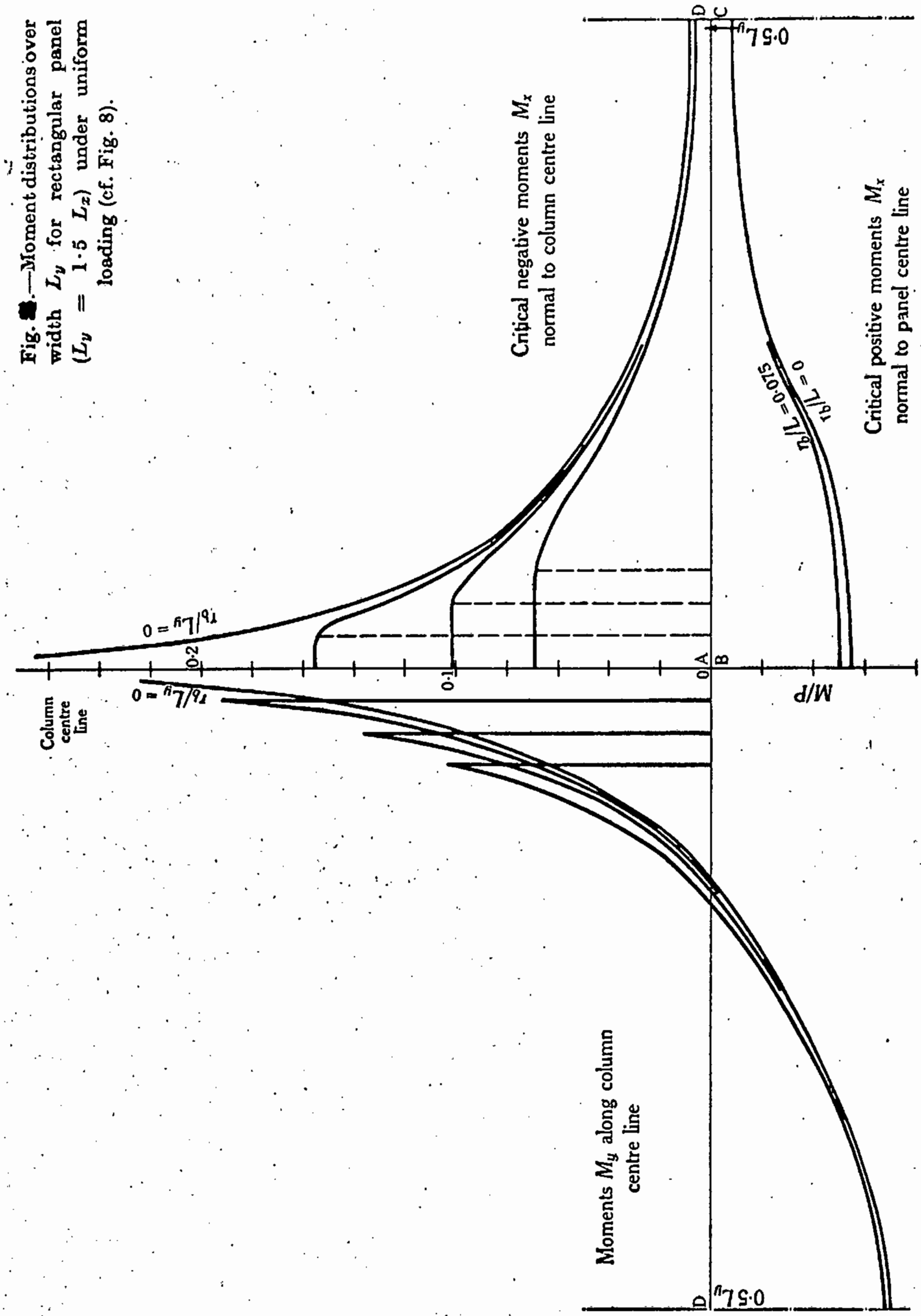


Fig. [redacted].—Moment distributions over width L_y for rectangular panel ($L_y = 1 \cdot 5\ L_x$) under uniform loading (cf. Fig. 8).

Fig. A.F9

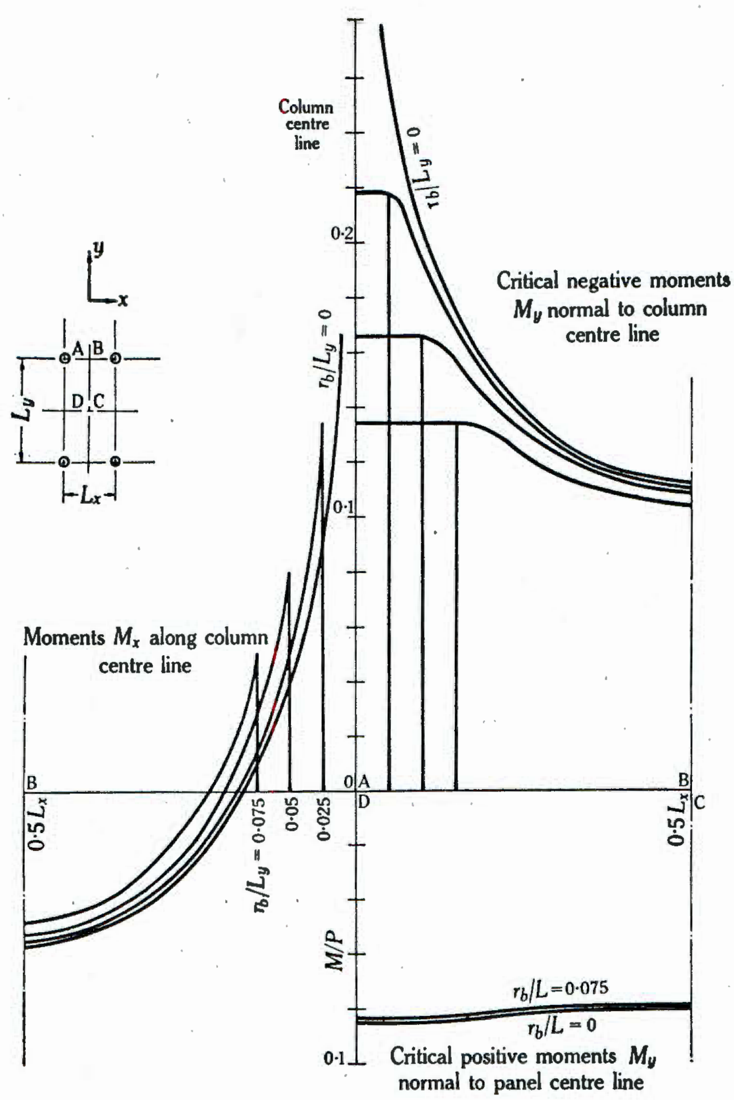


Fig. ■.—Moment distributions over width L_x for rectangular panel ($L_y = 2\,L_x$) under uniform loading (cf. Fig. 18).

Fig. A.F10

REINFORCEMENT DISTRIBUTION IN INTERNAL PANELS

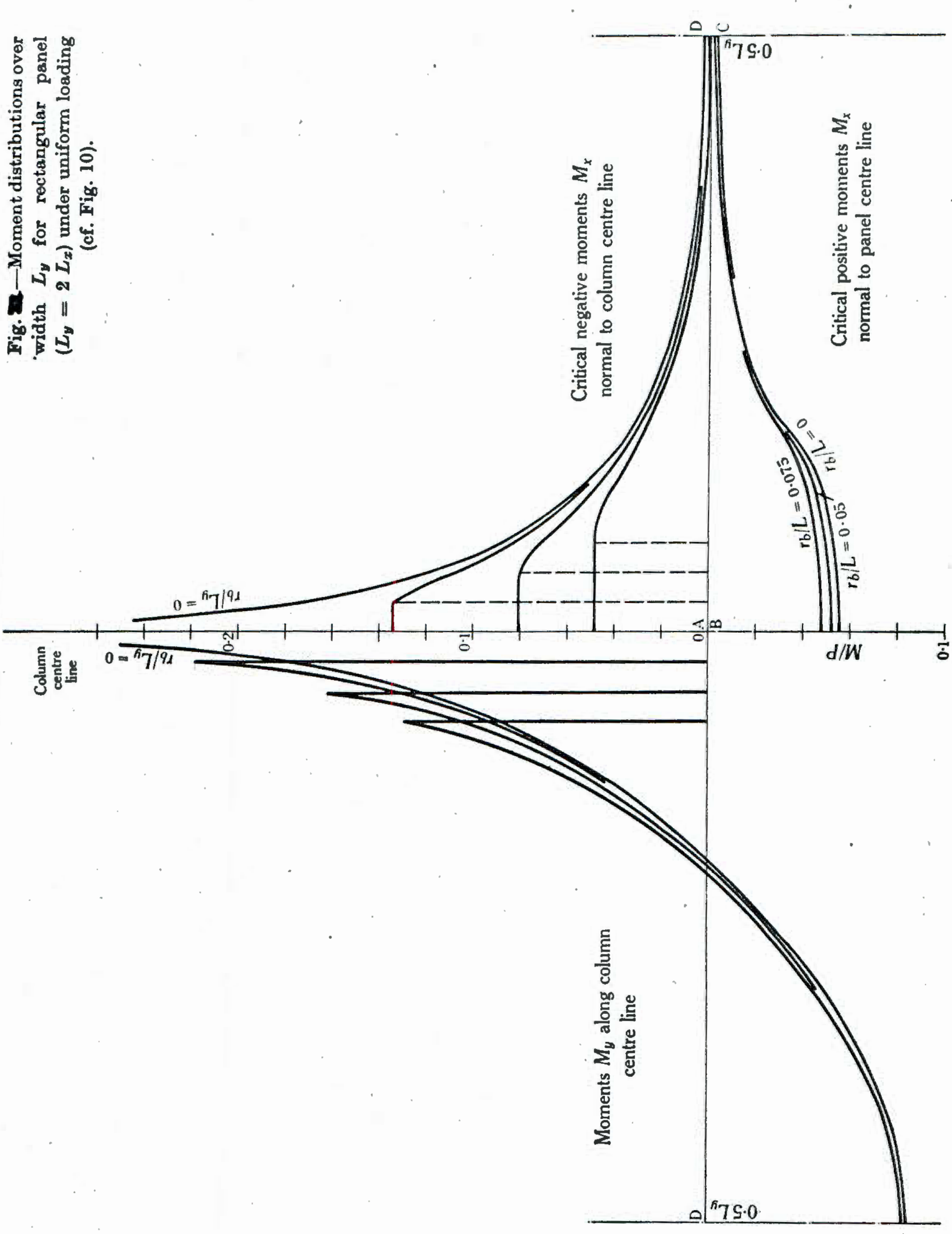


Fig. —Moment distributions over width L_y for rectangular panel ($L_y = 2\,L_x$) under uniform loading (cf. Fig. 10).

Fig. A.F11

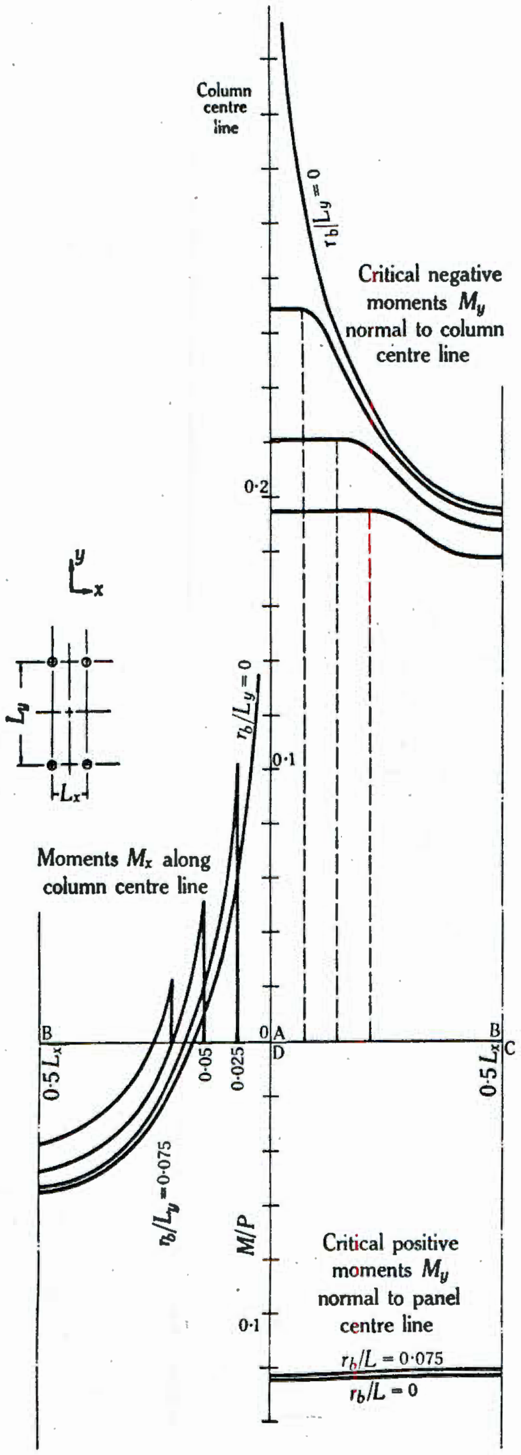


Fig. ■.—Moment distributions over width L_x for rectangular panel ($L_y = 3\ L_x$) under uniform loading (cf. Fig. 20).

Fig. A.F12

REINFORCEMENT DISTRIBUTION IN INTERNAL PANELS

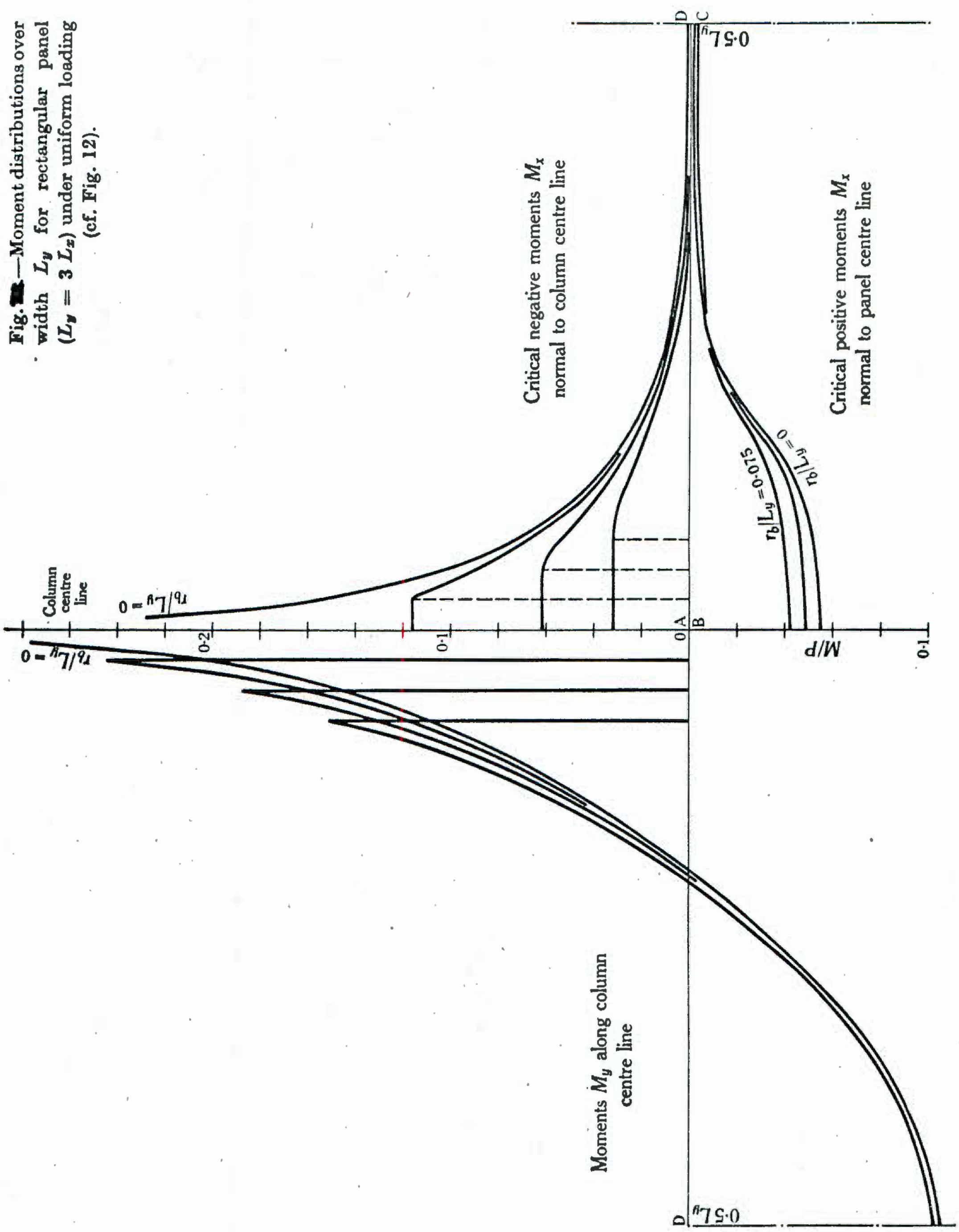


Fig. ■.—Moment distributions over width L_y for rectangular panel ($L_y = 3\,L_x$) under uniform loading (cf. Fig. 12).

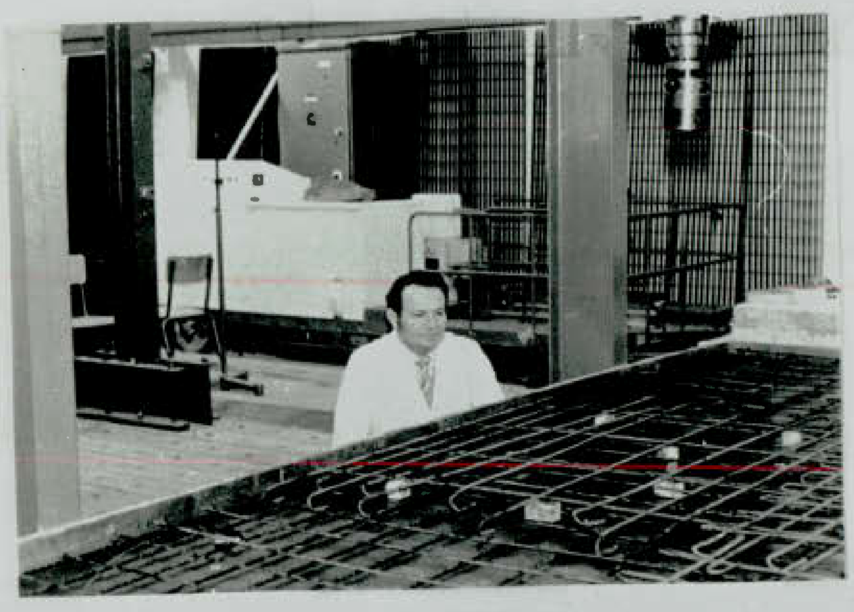


Fig. A.P1
Inspecting the Reinfmt.

Fig. A.P2 - Reinforcement in position before casting.

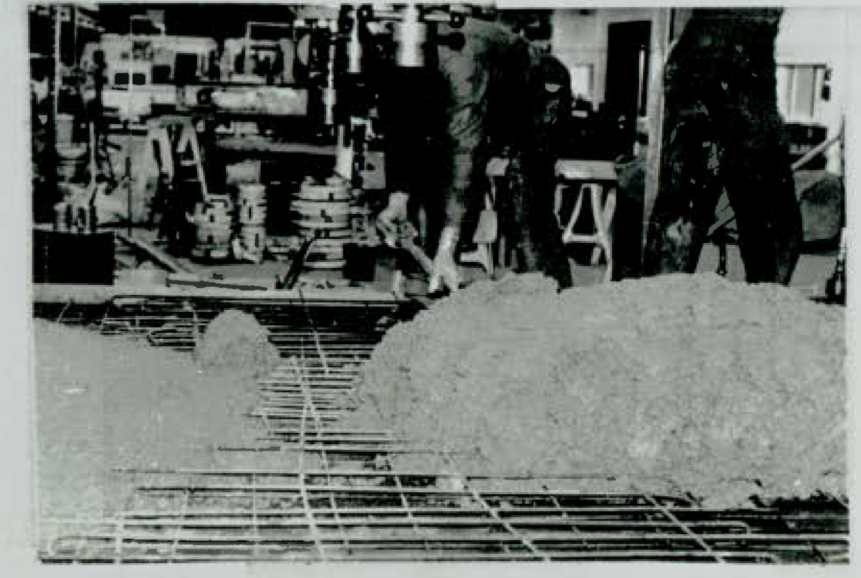
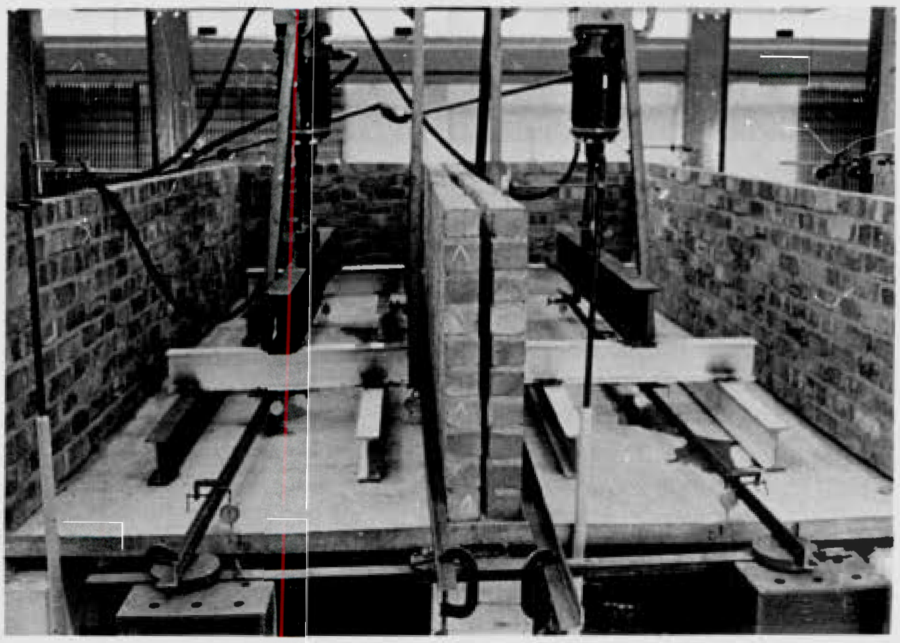


Fig. A.P3 - Casting the slab with a 50mm slump.



C F I

Fig. A.P4

Slab after failure, still supporting walls and loading rig.

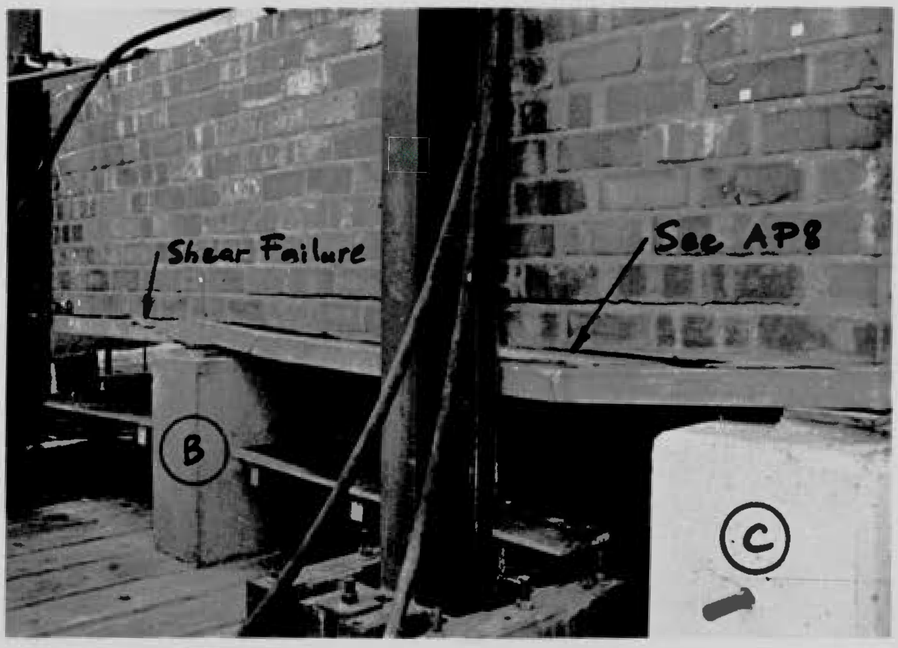


Fig. A.P.5 - Wall and Slab Edge A-B-C

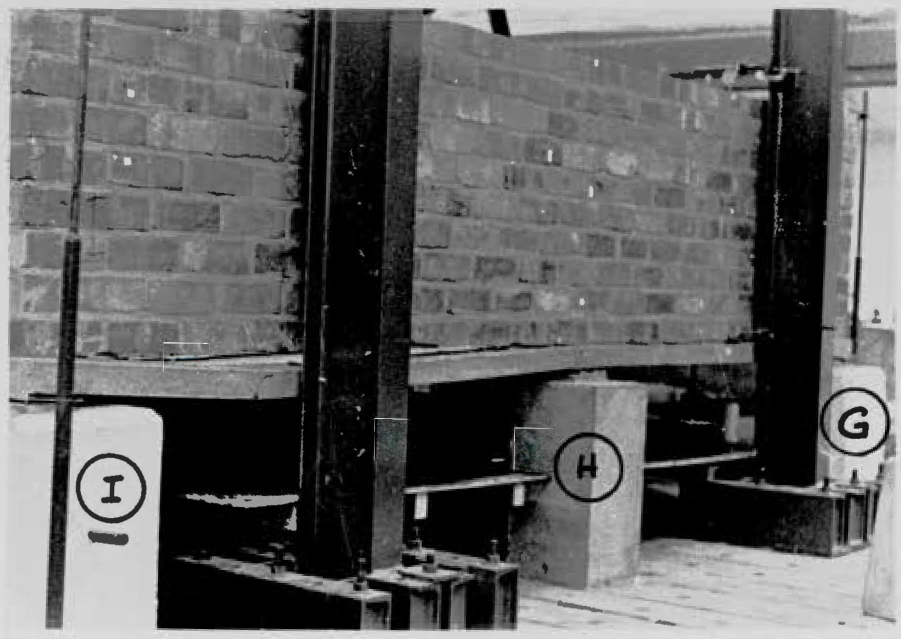
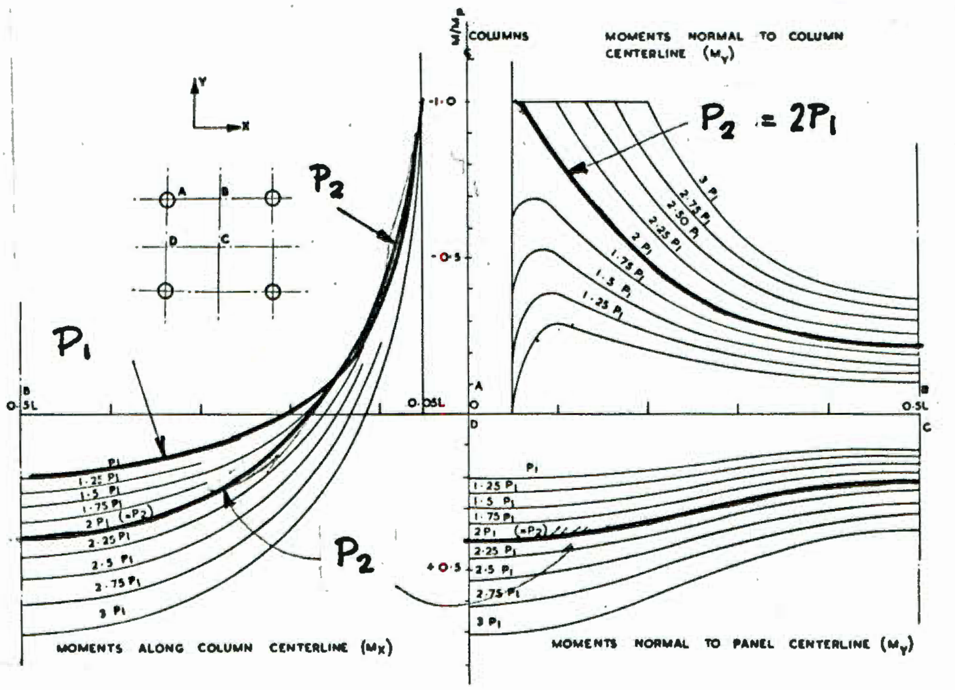


Fig. A.P6 - Wall and Slab Edge G-H-I

Fig. A.P7 - Cracks in top of slab and shear failure over B

Fig. A.P8- Wide Cracks in bottom of slab and spall in top under loaded areas



-Moments in a uniformly loaded square internal panel of a flat plate structure (in which M_p is constant, r_0 is $0.05L$, and $\nu = 0$) at various stages of yield. P_1 = maximum elastic load

Fig. A.F14